University of Alberta Library
0 1620 1602 1865

QA
14
C2
A436
2002
CURRGDHT

Ex libris
UNIVERSITATIS
ALBERTENSIS

Outcomes with Assessment Standards

for

Pure Mathematics 20

(August 2002)

This document is intended to assist teachers with the provincial implementation of Pure Mathematics 20.

This document replaces the February 2000 and September 2000 interim drafts and reflects the 2002 Program of Studies.

Alberta
LEARNING
Learning and Teaching Resources Branch

Alberta Learning Cataloguing in Publication Data

For more information, contact Alberta Learning, Learning and Teaching Resources Branch, 11160 Jasper Avenue, Edmonton, Alberta, Canada, T5K 0L2. Telephone: 780–427–2984; Fax: 780–422–0576. To be connected toll free inside Alberta, dial 310–0000.

The primary intended audience for this document is:

Administrators	
Counsellors	
General Audience	
Parent School Councils	
Parents	
Students	
Teachers	✓

Copyright ©2002, the Crown in Right of Alberta, as represented by the Minister of Learning. Alberta Learning, Learning and Teaching Resources Branch, 11160 Jasper Avenue, Edmonton, Alberta, Canada, T5K 0L2.

Permission is given by the copyright owner for any person to reproduce this document, or any part thereof, for educational purposes and on a nonprofit basis.

UNIVERSITY LIBRARY

ACKNOWLEDGEMENTS

This document was developed as a joint project of Alberta classroom teachers and staff at Alberta Learning. The cooperation of the following school jurisdictions is greatly appreciated.

Battle River Regional Division No. 31
Black Gold Regional Division No. 18
Buffalo Trail Regional Division No. 28
Calgary Roman Catholic Separate School District No. 1
Calgary School District No. 19
Christ the Redeemer Catholic Separate Regional Division No. 3
Clearview School Division No. 71
Edmonton Catholic Regional Division No. 40
Edmonton School District No. 7
Elk Island Catholic Separate Regional Division No. 41
Elk Island Public Schools Regional Division No. 14
Foothills School Division No. 38
Fort McMurray School District No. 2833
Fort McMurray Roman Catholic Separate School District No. 32
Golden Hills Regional Division No. 15
Grande Prairie School District No. 2357
Grande Yellowhead Regional Division No. 35
Greater St. Albert Catholic Regional Division No. 29
High Prairie School Division No. 48
Lakeland Roman Catholic Separate School District No. 150
Lethbridge School District No. 51
Medicine Hat School District No. 76
Northern Gateway Regional Division No. 10
Northern Lights School Division No. 69
Parkland School Division No. 70
Peace River School Division No. 10
Red Deer Catholic Regional Division No. 39
Red Deer School District No. 104
Rocky View School Division No. 41
Wetaskiwin Regional Division No. 11
Wild Rose School Division No. 66
Wolf Creek School Division No. 72
Yellowhead Tribal Council

The Alberta Learning team members were from the Curriculum Branch, Learner Assessment Branch, Learning and Teaching Resources Branch, and Learning Technologies Branch.

TABLE OF CONTENTS

Acknowledgements iii
Introduction 1
Purpose 1
Definitions and Terminology 1
Standards for Pure Mathematics 20 2
Assessment Plans, Suggested Weightings and Blueprints 5
Topics Related to Basic Resources 6
Technology Requirements 7
Mathematical Conventions Used in the Description of Sample Tasks 8
Standards Linked to Outcomes 9
Standards in Linear and Nonlinear Systems 11
Standards in Quadratic Functions and Equations 29
Standards in Polynomial and Other Nonlinear Equations and Functions 49
Standards in Formal Reasoning 75
Standards in Circles and Coordinate Geometry 93
Standards in Finance 113

INTRODUCTION

Pure Mathematics 20 was provincially implemented in September 1999. Teachers implementing this new course expressed a need for a common understanding of the curriculum and assessment standards. In response to this need, and in keeping with Alberta Learning's goal of establishing and effectively communicating clear outcomes and high standards for each area of learning, this standards document was developed.

PURPOSE

Outcomes with Assessment Standards for Pure Mathematics 20 contains samples of tasks linked to the specific outcomes from the program of studies, along with information and commentaries about standards. Its purpose is to provide teachers of Pure Mathematics 20 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices. This document is not intended as an assessment tool or test package, but only as a guide that provides models of tasks that can be assigned or discussed in the classroom. The sample tasks in this document are intended for teacher use, but they also can be used to communicate to the broader educational community examples of acceptable student work and excellent student work in Pure Mathematics 20.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating learning performance, the following standards apply:

- *curriculum* and *assessment standards* apply to the assessment of individual students
- *achievement standards* apply to the assessment of student populations.

In this document, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course or grade level of a program. The curriculum standards for Pure Mathematics 20 are defined by the general and specific outcomes outlined in the program of studies.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course or grade level of a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

Assessment Instrument

An *assessment instrument* is a group of questions or tasks given to students to ascertain if they have met the requirements of the acceptable standard or the standard of excellence.

Blueprint

A *blueprint* is a classification of the questions or tasks making up a particular assessment instrument or an assessment plan. The classification may be in terms of standards (acceptable or excellent), content sections (topics or titles), mathematical understandings (concepts, procedures, problem-solving skills), or any other such classification.

Conceptual Understanding

Tasks assessing *conceptual understanding* are characterized by the use of such action verbs as defining, demonstrating, describing, developing, establishing, explaining, giving, identifying, illustrating, linking, recognizing and representing.

Procedural Knowledge

Tasks assessing *procedural knowledge* are characterized by the use of such action verbs as approximating, calculating, constructing, estimating, factoring, locating, rationalizing, simplifying, sketching, solving, using and verifying.

Problem-solving Skills

Problem-solving skills refer to the construction of models describing applications in a broad range of contexts. They refer to the combination of algorithms to determine more complicated solutions and the use of appropriate technology to approximate difficult models or to carry out long calculation procedures. They also refer to the investigation of phenomena, the collection and interpretation of data in various contexts, and the transfer of knowledge from mathematics to other areas of human endeavour. Problem-solving tasks are characterized by the use of such action verbs as adapting, analyzing, combining, communicating, comparing, connecting, constructing, deriving, evaluating, fitting, investigating, justifying, modelling, proving, reconstructing and translating.

STANDARDS FOR PURE MATHEMATICS 20

The content specified in the program of studies for Pure Mathematics 20 contains the knowledge, skills and attitudes that all students are expected to acquire. As well, there are specific outcomes for problem solving and the use of technology. For complete details of the Pure Mathematics 20 course structure, refer to the program of studies.

Pure Mathematics 20 is designed to follow directly from Pure Mathematics 10, so students taking Pure Mathematics 20 are presumed to have reached the acceptable standard or better in the outcomes of Pure Mathematics 10. In particular, students will have facility in using the graphing calculator to graph functions; be able to use a variety of methods to add, subtract, multiply, divide and simplify polynomial and rational expressions; and have an ability to use coordinate methods to determine the properties of lines and line segments.

The assessment standards for Pure Mathematics 20 include an acceptable and an excellent level of performance. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts, and others which are nonroutine tasks in unfamiliar contexts.

Acceptable Standard

The *acceptable standard* of achievement in Pure Mathematics 20 is met by students who receive a course mark between, and including, 50 per cent and 79 per cent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined in the Pure Mathematics 20 program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Pure Mathematics 20 is met by students who receive a course mark at, or above, 80 per cent. Typically, these students have gained a breadth and depth of knowledge regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts. This standard signifies high-quality performance relative to the general and specific outcomes in the Pure Mathematics 20 program of studies.

Description of Standards

The following statements describe what is expected of Pure Mathematics 20 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard	*Standard of Excellence*
Students who meet the *acceptable standard* in Pure Mathematics 20 consistently perform acceptable work on routine and obvious tasks in familiar contexts.	Students who meet the *standard of excellence* in Pure Mathematics 20 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.
These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to pure mathematics at the 20 level, and apply what they know in daily living contexts.	These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to pure mathematics at the 20 level, apply what they know in daily living contexts, and provide alternative solution procedures to verify results.

To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts, and they construct mathematical models.	To meet the *standard of excellence,* students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts; and they construct mathematical models by translating words into suitable numbers, diagrams, tables, equations and variables.
Students meeting the *acceptable standard* apply what they know in solving straightforward problems in familiar settings and in analyzing simple mathematical models. They describe the steps they used to solve a particular problem, and verify and defend their solution to the problem.	Students meeting the *standard of excellence* apply what they know in solving routine and nonroutine problems in a broad range of settings. They describe the steps they used to solve a particular problem; defend their solution to the problem; and, where appropriate, provide alternative solution procedures to verify results.
Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.	Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

ASSESSMENT PLANS, SUGGESTED WEIGHTINGS AND BLUEPRINTS

These plans and blueprints provide general guidelines for the overall assessment of Pure Mathematics 20.

Suggested Weightings by Topic		
Topic	**Specific Outcomes (Summary)**	**Suggested Per Cent Weighting**
Linear and Nonlinear Systems	1.1 Graph linear inequalities, in two variables. 1.2 Solve systems of linear equations, in two variables. 1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. 1.4 Solve systems of linear equations, in three variables.	10–12%
Quadratic Functions and Equations	2.1 Determine the characteristics of the graph of a quadratic function. 2.2 Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. 2.3 Model real-world situations, using quadratic functions. 2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function. 2.5 Determine the character of the real and non-real roots of a quadratic equation.	13–15%
Polynomial and Other Nonlinear Equations and Functions	3.1 Solve nonlinear equations. 3.2 Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. 3.3 Perform operations on functions and compositions of functions. 3.4 Determine the inverse of a function. 3.5 Describe, graph and analyze polynomial and rational functions, using technology. 3.6 Formulate and apply strategies to solve absolute value equations, radical equations, rational equations, quadratic inequalities and polynomial inequalities.	25–28%
Formal Reasoning	4.1 Differentiate between inductive and deductive reasoning. 4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. 4.3 Use examples and counterexamples to analyze conjectures. 4.4 Distinguish between an "if–then" proposition, its converse and its contrapositive. 4.5 Prove assertions in a variety of settings, using direct reasoning.	10–12%
Circles and Coordinate Geometry	5.1 Use technology and measurement to confirm and apply circle properties. 5.2 Prove circle properties, using established concepts and theorems. 5.3 Solve problems, using a variety of circle properties, and justify the solution strategy used. 5.4 Solve problems involving distances between points and lines. 5.5 Verify and prove assertions in plane geometry, using coordinate geometry.	20–22%
Finance	6.1 Solve consumer problems. 6.2 Reconcile financial statements. 6.3 Solve budget problems, using graphs and tables to communicate solutions. 6.4 Plot and describe financial data of exponential form. 6.5 Solve investment and credit problems involving simple and compound interest.	10–12%

Suggested Per Cent Weightings by Standard and Skill				
	Conceptual Understanding	**Procedural Knowledge**	**Problem-solving Skills**	**Totals**
Acceptable Standard	≅20%	≅30%	≅25%	75–80%
Standard of Excellence	≅10%	≅5%	≅10%	20–25%
Totals	25–30%	35–40%	30–35%	100%

TOPICS RELATED TO BASIC RESOURCES

Topic	**Addison-Wesley** *Mathematics 11* (Western Canadian Ed.) **Chapters**	**McGraw-Hill Ryerson** *Mathpower 11* (Western Ed.) **Chapters**	**Distance Learning Pure Mathematics 20 (Module)**
Linear and Nonlinear Systems	5	1, 2	2
Quadratic Functions and Equations	2	3, 4	3
Polynomial and Other Nonlinear Equations and Functions	3, 4	4, 5	4
Formal Reasoning	6	6	5
Circles and Coordinate Geometry	7, 8, 9	7, 8	6 (Section 1 and Activity 1 of Section 2 only)
Finance	1	9	1

TECHNOLOGY REQUIREMENTS

To meet the outcomes of Pure Mathematics 20, students will need access to a graphing calculator and a computer with a spreadsheet program. In most classrooms, students will not use technology on a daily basis; however, there are some topics of study that require greater access than others. The following table contains descriptions of the technology-related knowledge and skills expected of students in each of the topic areas, with suggested time requirements.

Topic	**Estimated Time Requirement for Access to Technology**	**Technology-related Knowledge and Skills**
Linear and Nonlinear Systems	3–4 hours	***Students will be expected to:*** • graph an equation, using a graphing calculator – enter the equation in the $y =$ form – plot the graph • use the calculate function to estimate zeros, intersection points, and maximum and minimum points
Quadratic Functions and Equations	8–10 hours	• graph an equation, using a graphing calculator – enter the equation in the $y =$ form – plot the graph • display a table or use the trace function to estimate the domain and range • use the calculate function to estimate zeros, intersection points, and maximum and minimum points
Polynomial and Other Nonlinear Equations and Functions	15–17 hours	• graph an equation, using a graphing calculator – enter the equation in the $y =$ form – plot the graph • display a table or use the trace function to estimate the domain and range • use the calculate function to estimate zeros, intersection points, and maximum and minimum points
Formal Reasoning	none	• none
Circles and Coordinate Geometry	4–5 hours	• use dynamic geometry software to explore the properties of circles and polygons • use a compass, protractor and straightedge to explore the properties of circles and polygons
Finance	2–3 hours	• use a computer spreadsheet program, including the application of user-defined functions • use a financial solver program on the graphing calculator • use commercial budget software programs (optional)

MATHEMATICAL CONVENTIONS USED IN THE DESCRIPTION OF SAMPLE TASKS

Approximations, Estimates and Exact Values

An *approximation* to an exact value is a decimal representation, correct to several decimal places, used in a problem. Usually these approximations result from calculator usage. For example, $\sqrt{3}$ may be replaced by the approximation 1.7321… during the computational phase of a problem.

An *estimate* of an exact value is an approximation that can be derived mentally and used as part of a mental calculation. For example, the conjecture $\sqrt{2}+\sqrt{8}=\sqrt{10}$ can be shown to be false by using the estimates 1.5, 2.8 and 3.1 for $\sqrt{2}, \sqrt{8}$ and $\sqrt{10}$.

The *exact value* of a rational number is of the form $\frac{p}{q}$, where p and q are integers. Unless the decimal representation terminates, the decimal representation will be an approximation. So $-\frac{5}{3}$ is exact, while $-1.666\ldots$ is an approximation.

The *exact value* of an irrational number is expressed in such forms as sin 35°, $\sqrt{3}$ or $\frac{\pi}{6}$. The decimal representation will always be an approximation. So sin 35° is exact, while 0.573576… is an approximation.

Calculator Use in Multipart Problems

The general rule is to keep the calculator running from beginning to end, using all the available decimal places. All answers, both intermediate and final, are reported to the appropriate number of significant figures. However, rounded intermediate answers are **not** to be used as inputs into subsequent calculations. For example, a right-angled triangle with a measured hypotenuse of 11.9 cm and a shortest side of 6.1 cm would have the third side reported as 10.2 cm. However, if the length of the third side were used in the second part of a calculation, the more precise value 10.21763182… would be used as the input value.

Measured and Exact Input Data

If the *input data is measured*, all output data should be recorded in decimal form to the same quality as the least reliable part of the input data. For example, if the input data had 57.235 g for the mass and 17 mL for the volume, the density would be recorded as 3.4 g/mL with two significant digits. As a matter of convention, trigonometric problems with lengths to three significant digits yield angles to one-tenth of a degree, or one-hundredth of a radian.

If the *input data is exact,* all output data should be recorded in fractional form if at all possible. For example, the slope of the line joining the points (6, 11) and (17, 32) should be recorded as $\frac{21}{11}$, which is exact, rather than the decimal form 1.9090….

For Pure Mathematics 20, all trigonometric data can be assumed to be measured, together with most of the estimated probabilities used in calculations of expectation values. All input data in coordinate geometry, algebraic expressions, relations and functions can be assumed to be exact. Most of the input data in number tables and number patterns can be assumed to be exact, except that money outputs are rounded to the nearest dollar or the nearest cent, depending on the context.

If there is any doubt whether any item of input data is exact or measured, assume it to be exact if it is part of a Pure Mathematics 20 problem.

STANDARDS LINKED TO OUTCOMES

The remainder of this document consists of several components that, taken together, provide guidance to teachers about the scope, depth and breadth of Pure Mathematics 20.

Topics

Topics are the mathematical themes that are used to organize concepts. The general and specific outcomes and sample tasks in this part of the document are grouped according to topics.

General and Specific Outcomes

The general and specific outcomes for Pure Mathematics 20 identify what students are expected to know and be able to do upon completion of the course.

Strands

Strands are the underlying themes running throughout all of the mathematics programs. The Pure Mathematics 20 course includes:

- Number (Number Operations)
- Patterns and Relations (Patterns)
- Patterns and Relations (Variables and Equations)
- Patterns and Relations (Relations and Functions)
- Shape and Space (Measurement)
- Shape and Space (3-D Objects and 2-D Shapes)

Mathematical Processes

These are the critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. One or more of the following mathematical processes is emphasized for each specific outcome.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

More information on the general and specific outcomes, strands, and mathematical processes can be found in the *Mathematics Applied and Pure Programs Program of Studies*.

Notes

Each specific outcome is followed by a note that provides additional information about the intended depth and breadth for that outcome. Teachers can refer to these for guidance when planning classroom activities and assessments that address the specific outcome at an appropriate level for the course.

Sample Tasks

Linked to each specific outcome are sample tasks. These consist of questions, problems or activities for classroom use with students. Teachers may use these directly or as models for developing their own tasks for group or independent work in the classroom, or as part of formative and summative assessment of students. The sample tasks and suggested solutions were developed and validated by classroom teachers of Pure Mathematics 20, but **they have not been validated with students**. They represent models of the kinds of questions and problems students should be able to solve, and the kinds of activities students should be able to perform to meet the specific outcome to which they are linked.

The following two examples illustrate the components expected in a student's written response to a problem, when a graphing calculator has been used to assist in determining the answer.

Example 1: When asked to solve a problem that asks for the coordinates of key points, from a description of a situation that can be expressed as a function, the following should be included in the student's response:

- the function in $y =$ form
- a sketch of the graph with the scales labelled
- the key points identified on the sketch
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Example 2: When solving a problem that involves using the regression menu on a graphing calculator to determine the line of best fit for a data set, students can be expected to include the following in the written response:

- the data from input lists
- the regression formula used
- the equation of the line of best fit
- a sketch of the graph with the scales labelled
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Descriptions of Student Performance (Related to Specific Outcome)

Each specific outcome is followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions represent the judgements of practising classroom teachers in regard to how their students would perform on the particular specific outcome.

Descriptions of Student Performance (Related to Sample Tasks)

The sample tasks for a particular specific outcome are followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions, though **not based on actual student responses**, represent the judgements of practising classroom teachers in regard to how their students would perform, given these particular tasks.

STANDARDS IN LINEAR AND NONLINEAR SYSTEMS

GENERAL OUTCOME

- Represent and analyze situations that involve expressions, equations and inequalities.

SPECIFIC OUTCOMES

1.1 Graph linear inequalities, in two variables. [PS, V]

1.2 Solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]

1.4 Solve systems of linear equations, in three variables:
- algebraically
- with technology.

[CN, PS, T, V]

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.1 Graph linear inequalities, in two variables. [PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 3.1.
- Students were introduced to inequalities with one variable in Grade 9 mathematics, where they solved inequalities and graphed inequalities on a number line.
- Coordinate geometry skills are essential for this specific outcome. The concepts of plotting points and intercepts, line graphing, and the use of calculators are the more important concepts to review.
- $Ax + By + C = 0$ can be sketched using intercepts.
- Conversion from the $Ax + By + C = 0$ form to any $y =$ form is a necessary preliminary step to the use of a graphing calculator.
- Window settings, on the graphing calculator, are useful to replace the plotting of horizontal and vertical lines. For example, $x < 5$ could be entered as a window setting of $x_{\max} = 5$.
- Students should first graph manually—to solve for and sketch the solution region—and then graph using the graphing calculator. An even balance between both approaches is recommended.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• graph the boundary line between two half planes • use a test point, usually (0, 0), to determine the solution region that satisfies the inequality, given a boundary line • graph a linear inequality expressed in the form $y = mx + b$, using $<, >, \leq, \geq$ • rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, where A, B, C are integral and $B > 0$.	• distinguish between the use of solid and broken lines in solution regions • graph any linear inequality in two variables • rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, and graph • explain why the shaded half plane represents the solution region of the inequality.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.1 Graph linear inequalities, in two variables. [PS, V]

Sample Tasks

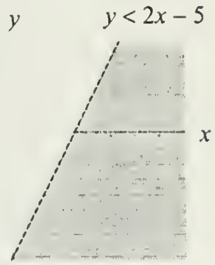
[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Graph $y < 2x - 5$.

Solution:

1.

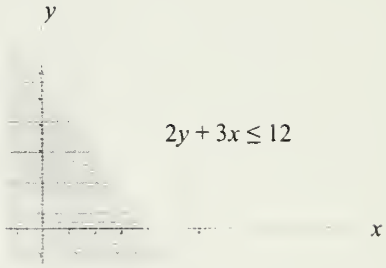


Question:

2. Graph $2y + 3x \leq 12$.

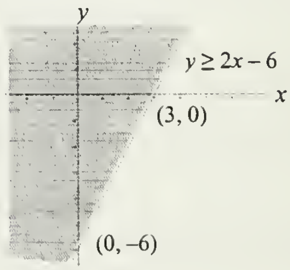
Solution:

2.



Question:

3. Given the following graph, provide the inequality.



Solution:

3. $y \geq 2x - 6$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.1 Graph linear inequalities, in two variables. [PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

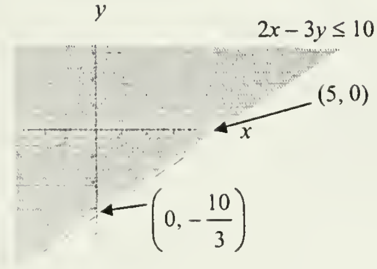
(continued)

Question:

4. Graph $2x - 3y \leq 10$ with a graphing calculator.

Solution:

4. $y \geq \frac{2}{3}x - \frac{10}{3}$



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete questions 1 and 2 on graph paper or on a graphing calculator, but may use solid lines rather than broken lines for the boundaries • determine the x-intercept as (5, 0) in question 4.	• complete questions 1, 2 and 3, including the boundaries • complete question 4.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.2 Solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 3.2.
- Students should recognize the graphical significance of one or many solutions, or no solution.
- Solving systems of linear inequalities is not part of this specific outcome.
- Review equations, if specific outcome 1.2 is to be completed before specific outcome 1.1.
- Addison-Wesley *Mathematics 11* includes both linear and nonlinear systems in Chapter 5. McGraw-Hill Ryerson *Mathpower 11* keeps linear and nonlinear systems as separate chapters.
- Specific outcome 1.3 introduces nonlinear systems, and specific outcome 1.4 includes systems in three variables.
- Students should be exposed to problems in a variety of real-life contexts.
- Rearranging systems to the $y =$ form and then solving graphically is not an algebraic solution.

<table>
<tr><th colspan="2">Descriptions of Student Performance (Related to Specific Outcome)</th></tr>
<tr><td>A student demonstrating the <i>acceptable standard</i> can:</td><td>A student demonstrating the <i>standard of excellence</i> can also:</td></tr>
<tr><td>• solve a basic system of equations, using:
– an elimination solution that requires only one equation to be multiplied by a scalar factor
– a substitution solution when one equation has a variable with a coefficient of one
• give an approximation to a graphical solution
• select a preferred method to solve all systems
• state the solution set for a system of parallel lines
• solve word problems that require a system of equations.</td><td>• vary the window when completing the graphical solution
• complete an elimination solution when it is necessary to multiply both equations by a scalar factor
• state the solution set for a system of coincident lines
• apply the most appropriate method for solving a system efficiently.</td></tr>
</table>

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.2 Solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

Question:

1. Solve algebraically:
   $2x + 3y = 12$
   $x - y = 1$

Solution:

Elimination

1. $(x - y = 1) \times 2$
   $2x - 2y = 2$

$$\begin{aligned} 2x + 3y &= 12 \\ -\,\underline{(2x - 2y = \;\;2)} & \\ 0x + 5y &= 10 \\ \frac{5y}{5} &= \frac{10}{5} \\ y &= 2 \end{aligned}$$

Substitute for y:
$x - 2 = 1$
$x = 3$

The solution is $x = 3$, $y = 2$.

Substitution

1. Substitute $x = y + 1$ into $2x + 3y = 12$.

$$\begin{aligned} \text{Then } 2(y + 1) + 3y &= 12 \\ 5y + 2 &= 12 \\ y &= 2 \end{aligned}$$

$x = y + 1$
So $x = 3$
The solution is $x = 3$, $y = 2$.

Question:

2. Solve algebraically:
   $2x + 3y = 12$
   $5x + 2y = -3$

Solution:

2. $(2x + 3y = 12) \times 2$
   $4x + 6y = 24$

$(5x + 2y = -3) \times 3$
$15x + 6y = -9$

$$\begin{aligned} 15x + 6y &= -9 \\ -\,\underline{(4x + 6y = 24)} & \\ 11x + 0y &= -33 \\ \frac{11x}{11} &= -\frac{33}{11} \\ x &= -3 \end{aligned}$$

Substitute for x:

$$\begin{aligned} 5(-3) + 2y &= -3 \\ -15 + 2y &= -3 \\ \frac{2y}{2} &= \frac{12}{2} \\ y &= 6 \end{aligned}$$

The solution is $x = -3$, $y = 6$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.2 Solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

Sample Tasks

☐ Conceptual
☐ Procedural
☑ Problem-solving

(continued)

Question:

3. Three pencils and four erasers cost \$2.50, while one pencil and three erasers cost \$1.25. Find the cost of a pencil and the cost of an eraser.

Solution:

3. Let the cost of a pencil = P.
Let the cost of an eraser = E.
$3P + 4E = \$2.50$ and $1P + 3E = \$1.25$.

$$(1P + 3E = \$1.25) \times 3$$
$$3P + 9E = \$3.75$$

Subtract:

$$\begin{array}{r} 3P + 9E = \$3.75 \\ -\underline{(3P + 4E = \$2.50)} \\ \dfrac{5E}{5} = \dfrac{\$1.25}{5} \\ E = \$0.25 \end{array}$$

Substitute for E:

$$\begin{aligned} 3P + 4E &= \$2.50 \\ 3P + 4(0.25) &= \$2.50 \\ \frac{3P}{3} &= \frac{\$1.50}{3} \\ P &= \$0.50 \end{aligned}$$

Each pencil costs \$0.50.
Each eraser costs \$0.25.

Question:

4. Jojo wants to set up an Internet connection with Easy Shop Internet. He has two plans to choose from. Plan A costs \$30.00 per month and includes a user fee of \$0.10 per minute. Plan B costs \$10.00 per month and includes a user fee of \$0.15 per minute.
   a. What would determine which plan is the most economical?
   b. When would both plans be equally economical to use? Using the given information, find the solution graphically. Draw the image that appears on your graphing calculator screen, and fill in the blanks below.

Y_1 =
Y_2 =
X min =
X max =
Y min =
Y max =

   c. Find the solution algebraically by either substitution or elimination.
   d. Are the results in b and c the same? Explain.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.2 Solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

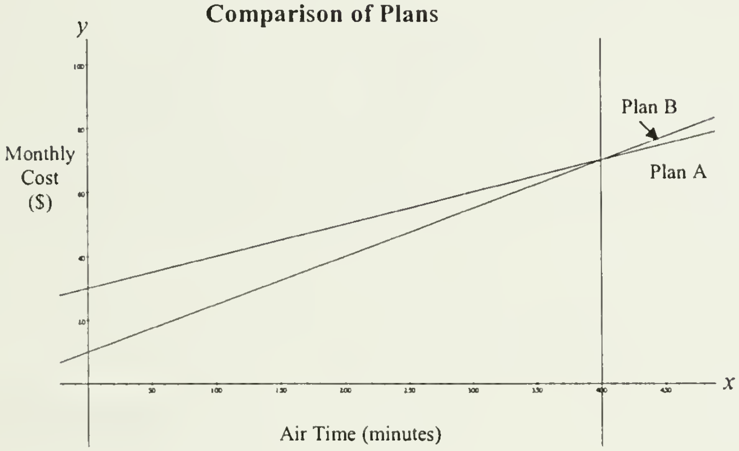
Solution:

4. a. The number of minutes of use should be used to determine which is most economical.

b.

$Y_1 = 30 + 0.10\ X$
$Y_2 = 10 + 0.15\ X$
X min = 380
X max = 420
Y min = 68
Y max = 72

The solution is (400, 70). This means at 400 minutes usage, the cost of both plans is the same.


Comparison of Plans

c. $30 + 0.1x = y$ and $y = 10 + 0.15x$; therefore,

$$30 + 0.1x = 10 + 0.15x$$
$$20 = 0.05x$$
$$400 = x$$

Each plan costs $70 for 400 minutes usage.

d. The solution may not be exactly the same, because the graph is always an approximation.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve question 1, using a preferred method that may not be the most efficient or appropriate • solve question 2, both algebraically and graphically • graph the equations and find the point of intersection for question 4.	• use the most appropriate method; i.e., substitution, to solve question 1 • solve systems of equations algebraically when the coefficients are any integers or rational numbers, as in questions 3 and 4.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome has a strong relationship and some commonality with specific outcomes 1.2 and 3.1.
- Limit equations to functions. Restrict examples to two equations with two unknowns.
- Algebraic verification is required; algebraic solutions to a limited range of examples are optional.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• graph the system, and use the graphing calculator to derive an approximate solution • verify solutions to equations algebraically.	• recognize that the solution obtained on the calculator may not be the exact solution • solve systems algebraically, using elimination or substitution • identify additional solutions outside the viewing window.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]

Sample Tasks

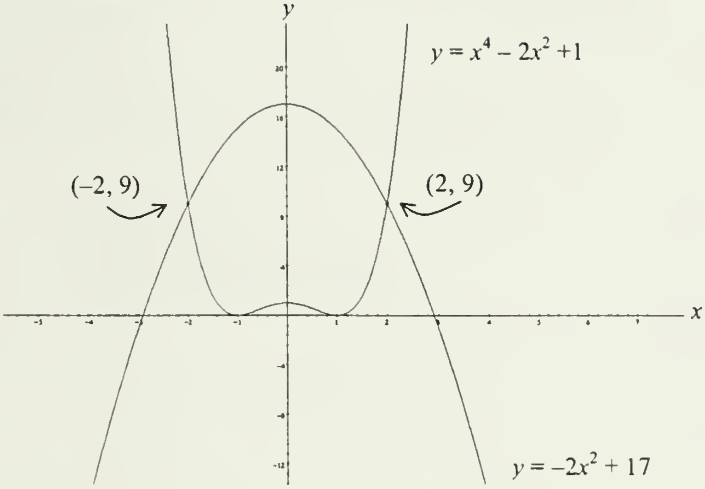
☐ Conceptual
☑ Procedural
☐ Problem-solving

Question:

1. Solve the following system of equations.
$y = x^4 - 2x^2 + 1$
$y = -2x^2 + 17$

Solution:

1. (graphical solution)



(algebraic solution)

$$x^4 - 2x^2 + 1 = -2x^2 + 17$$
$$x^4 = 16$$
$$x = \pm 2$$

$y = -2(2)^2 + 17 = 9$
or
$y = -2(-2)^2 + 17 = 9$
Intersection points: (2, 9) and (−2, 9).

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

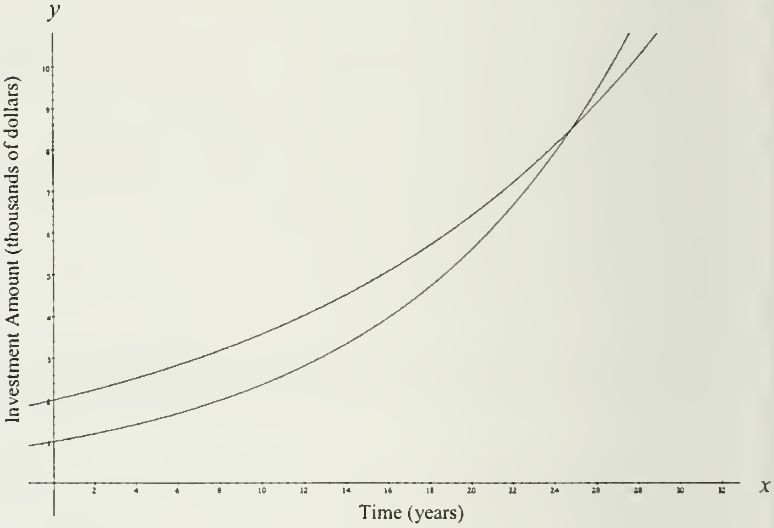
(continued)

Question:

2. When does an investment of $1000, invested at 9% per year, compounded yearly, amount to more than an investment of $2000, invested at 6% per year, compounded yearly?

Solution:

2. The $1000 investment will grow according to the equation $A(t)=(1.09)^t$, and the $2000 investment will grow according to the equation $A(t)=2(1.06)^t$. The graphs are shown in the diagram below.



The intersection point is (24.84, 8502). However, the context requires whole numbers of years, so it will take 25 years before the $1000 investment is worth more than the $2000 investment.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

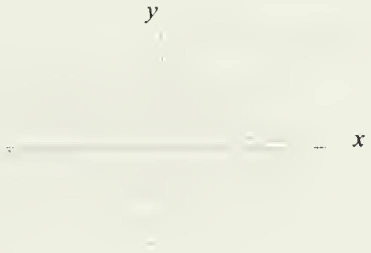
(continued)

Question:

3. Consider the following system:

$y = \dfrac{6}{x}$ and $y = mx + b$,

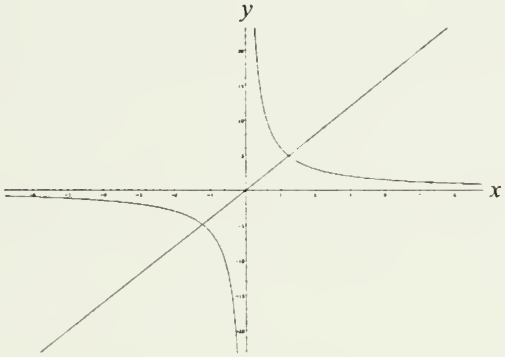
where m and $b \in R$.



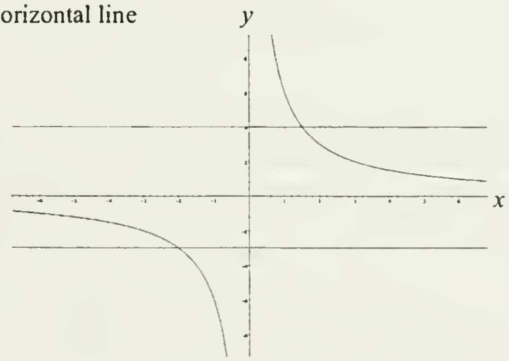
Find the number of solutions for each of the following cases.
a. $m = 4$, $b = 0$
b. $m = 0$
c. m is undefined
d. $m > 0$, $b =$ anything
Draw a sample line for each solution.

Solution:

3. a. two



b. zero or one; with zero for the line $y = 0$ and one for any other input horizontal line



(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]

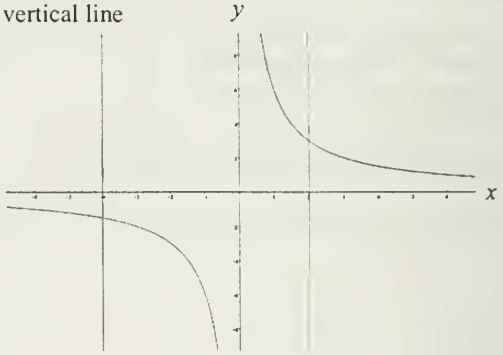
Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

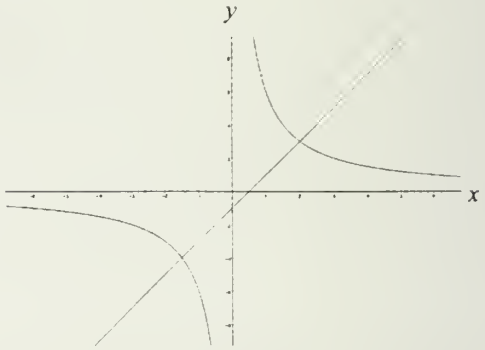
(continued)

Solution:

3. c. zero or one; with zero for the line $x = 0$ and one for any other input vertical line



d\. two



Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• graph the system in question 1, and determine the solutions (2, 9) and (−2, 9) easily from the graph without changing the window • determine the point of intersection in question 2, if the equations are given • partially solve question 3, indicating the implications of $m > 0$ and that $y = 6/x$ approaches, but does not cross, the x and y axes • provide answers to question 3, parts a, b and c, based upon specific values of m and b.	• determine the point of intersection in question 2, if the equations are not given • round the final answer in question 2 • determine that the answers to question 3, parts b and c, may vary according to the value of the x- or y-intercept of the line • complete question 3, part d.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.4 Solve systems of linear equations, in three variables:
- algebraically
- with technology.

[CN, PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- It is advisable to use an algebraic approach to explore this concept first, as it is critical to the understanding of a technological solution.
- One algebraic method is to change the three-variable system to a two-variable system by eliminating a variable. Then problems can be solved using the methods from specific outcome 1.2.
- Three practical ways to use technology to solve a three-variable system are to use:
  - a spreadsheet document
  - the matrix algebra function on the calculator
  - the *LINSYST3* program from the Addison-Wesley *Mathematics 11* teacher resource book.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate a system of equations from a problem • solve a system where either: – some variables have zero as a coefficient – elimination can be accomplished without scalar multiplication.	• solve a system where: – scalar multiplication is involved – no variables have zero as a coefficient.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.4 Solve systems of linear equations, in three variables:
- algebraically
- with technology.

[CN, PS, T, V]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

Question:

1. Solve this system of equations.
$x + y - z = 10$ ①
$x - y + z = 4$ ②
$x - 2y + z = 9$ ③

Solution:

Add ① and ②:	Subtract ③ from ②:	Substitute into ①:
$2x = 14$		$7 - 5 - z = 10$
$x = 7$	$y = -5$	$z = -8$

The solution is $x = 7$, $y = -5$ and $z = -8$.

Question:

2. Solve this system of equations.
$x + 3y + 4z = 19$ ①
$x + 2y + z = 12$ ②
$x + y + z = 8$ ③

Solution:

2. Subtract ② from ①:	Subtract ③ from ②:	Substitute from ⑤ into ④
$x + 3y + 4z = 19$	$x + 2y + z = 12$	$4 + 3z = 7$
$\underline{-(x + 2y + z = 12)}$	$\underline{-(x + y + z = 8)}$	$\frac{3z}{3} = \frac{3}{3}$
$y + 3z = 7$ ④	$y = 4$ ⑤	$z = 1$ ⑥

Substitute from ⑤ and ⑥ into ①:

$$\begin{aligned} x + 3(4) + 4(1) &= 19 \\ x + 12 + 4 &= 19 \\ x + 16 &= 19 \\ x &= 3 \end{aligned}$$

The solution is $x = 3$, $y = 4$ and $z = 1$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

1.4 Solve systems of linear equations, in three variables:
- algebraically
- with technology.

[CN, PS, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

3. Students were asked about the language courses they took in high school. The results of the survey and the total enrollment in each course were as follows.

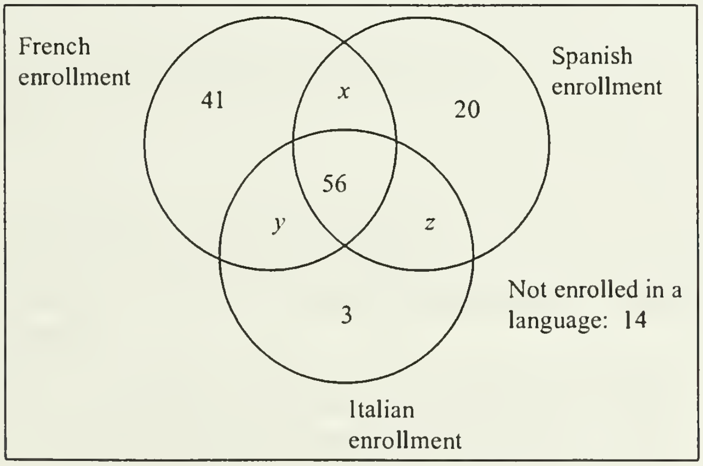
Language Courses Taken	Number of Students
French only	41
Spanish only	20
Italian only	3
All three: French, Spanish and Italian	56
Not taking a language	14

Total enrollment in French:	131 students
Total enrollment in Spanish:	115 students
Total enrollment in Italian:	82 students

a. Draw a Venn diagram to summarize this data.
b. The survey did not ask students if they were taking two of the three language courses. Determine these three unknown values.
c. How many students were surveyed?

Solution:

3. a.



b. Let x be the number of students enrolled in both French and Spanish.
Let y be the number of students enrolled in both French and Italian.
Let z be the number of students enrolled in both Spanish and Italian.

(continued)

General and Specific Outcomes
General Outcome Represent and analyze situations that involve expressions, equations and inequalities. **Specific Outcome** 1.4 Solve systems of linear equations, in three variables: • algebraically • with technology. [CN, PS, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution:

3. b. Then
French enrollment = $x + y + 41 + 56 = 131$
Spanish enrollment = $x + z + 20 + 56 = 115$
Italian enrollment = $y + z + 3 + 56 = 82$

So:
$x + y = 34$
$x + z = 39$
$y + z = 23$

And:
$y - z = -5$
$y + z = 23$

Therefore:
$x = 25$
$y = 9$
$z = 14$

c. Total surveyed:

$41 + 56 + 20 + 3 + 14 + 25 + 9 + 14 = 182$ students.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve a three-variable system of equations, such as in question 1, where most coefficients are small integers and two variables can be eliminated in one step • solve question 2, as there is no scalar multiplication to eliminate both x and z; using the second and third equations and substituting the value of y into the equations makes the original question into a two-equation, two-variable system • solve question 3 using technology, or provide a partial algebraic solution.	• solve question 3 completely, using an algebraic method.

STANDARDS IN QUADRATIC FUNCTIONS AND EQUATIONS

GENERAL OUTCOME

- Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

SPECIFIC OUTCOMES

2.1 Determine the following characteristics of the graph of a quadratic function:
  - vertex
  - domain and range
  - axis of symmetry
  - intercepts.

[C, PS, T, V]

2.2 Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]

2.3 Model real-world situations, using quadratic functions. [CN, PS]

2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
  - factoring
  - the quadratic formula
  - graphing.

[CN, E, T, V]

2.5 Determine the character of the real and non-real roots of a quadratic equation, using:
  - the discriminant in the quadratic formula
  - graphing.

[C, R, T, V]

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.1 Determine the following characteristics of the graph of a quadratic function:
- vertex
- domain and range
- axis of symmetry
- intercepts.

[C, PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 2.2.
- Encourage students to find the maximum/minimum values and zeros, using the calculate menu on a graphing calculator. Note: Using the trace function only approximates the solution.
- Set notation for domain and range is not necessary. Relate range to the maximum or minimum values of the function.
- Function notation has been covered in Pure Mathematics 10, so students will recognize $f(t)$ as being equivalent to y.
- To save time, specific outcome 2.2 can be incorporated into this specific outcome.
- Students should be able to determine the characteristics, given either the graph or the equation.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can: • modify the viewing window so the characteristics of the graph can be seen, if the characteristics cannot be obtained readily from the standard viewing window • estimate the domain and range of a graph, using the standard viewing window • identify the axis of symmetry for a graph • write the equation for the axis of symmetry.	There are no particular expectations at the standard of excellence for this specific outcome.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.1 Determine the following characteristics of the graph of a quadratic function:
- vertex
- domain and range
- axis of symmetry
- intercepts.

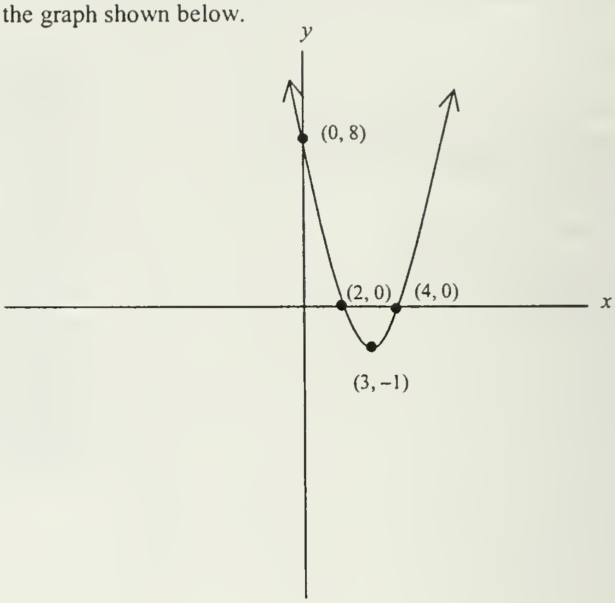
[C, PS, T, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Determine the vertex, domain, range, axis of symmetry and intercepts of the graph shown below.



Solution:

1. vertex: $(3, -1)$
domain: $x \in R$
range: $y \geq -1$
axis of symmetry: $x - 3 = 0$
x-intercepts: 2, 4
y-intercept: 8

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.1 Determine the following characteristics of the graph of a quadratic function:
- vertex
- domain and range
- axis of symmetry
- intercepts.

[C, PS, T, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

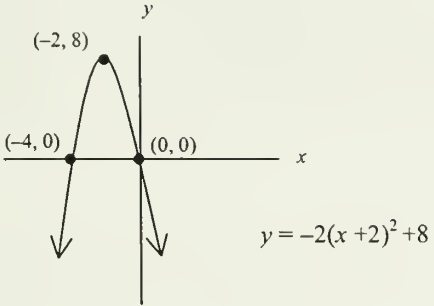
(continued)

Question:

2. Use your graphing calculator to graph $y = -2(x+2)^2 + 8$; and determine the vertex, domain and range, axis of symmetry, and intercepts.

Solution:

2.



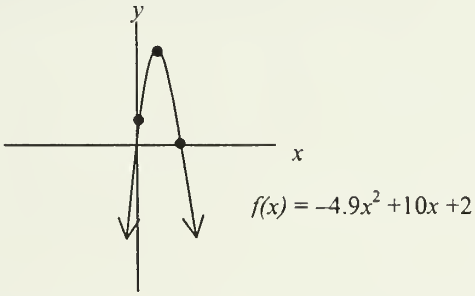
Vertex: $(-2, 8)$
Domain: $\{x \in R\}$ Range: $\{y \leq 8, y \in R\}$
Axis of symmetry: $x + 2 = 0$, or $x = -2$
x-intercepts: 0, –4; y-intercept: 0

Question:

3. Use your graphing calculator to graph $f(x) = -4.9x^2 + 10x + 2$; and determine the vertex, domain and range, axis of symmetry, and intercepts. Round to the nearest tenth.

Solution:

3.



Vertex: $(1.0, 7.1)$
Domain: $\{x \in R\}$ Range: $\{y \leq 7.1, y \in R\}$
Axis of symmetry: $x - 1.0 = 0$, or $x = 1.0$
x-intercepts: –0.2, 2.2; y-intercept: 2

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can: • provide solutions to questions 1, 2 and 3.	There are no particular expectations at the standard of excellence for these sample tasks.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.2 Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Students need to be able to communicate how to transform the quadratic function from $y = ax^2 + bx + c$ to $y = a(x - p)^2 + q$.
- Explore values of a, p and q; and communicate how those values affect the graph of $y = a(x - p)^2 + q$.
- Students should be able to identify the maximum or minimum values for the function from the completed square form.
- This specific outcome is related to specific outcome 2.1.
- When completing the square, the process should be emphasized; students are expected to show their work.
- Teachers may want to develop the vertex formula: $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$, or $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
- Students will need to become knowledgeable about terms related to transformations; e.g., reflection, translation, expansion, contraction, stretch.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete the square when a is a positive integer and p is an integer • determine the values of p and q, when given the graph of the function.	• complete the square when a and p are any rational numbers • determine the value of a, when given the graph of the function.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.2 Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Express $y = 2x^2 + 12x + 1$ in completed square form.

Solution:

1. $y = 2(x^2 + 6x) + 1$

$y = 2(x^2 + 6x + 9) - 18 + 1$

$y = 2(x + 3)^2 - 17$

Question:

2. Express $y = 3x^2 + 6x + m$ in completed square form.

Solution:

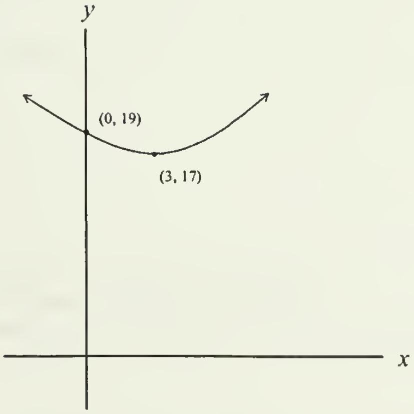
2. $y = 3(x^2 + 2x) + m$

$y = 3(x^2 + 2x + 1) + m - 3$

$y = 3(x + 1)^2 + m - 3$

Question:

3. Given the following graph, write the equation in the form $y = a(x - p)^2 + q$.



(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.2 Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution:

3. $y = a(x - p)^2 + q$

Vertex: (3, 17)

$y = a(x - 3)^2 + 17$

Passes through the point (0, 19)

$19 = a(0 - 3)^2 + 17$

$19 = a(9) + 17$

$2 = 9a$

$a = \frac{2}{9}$

$y = \frac{2}{9}(x - 3)^2 + 17$

[✓] Conceptual
[✓] Procedural

Question:

4. The following three transformations are applied, in order, to the graph of $y = x^2$: a reflection about the x-axis, a vertical stretch of scale factor 2, a horizontal translation of 2 to the right and a vertical translation of –4. At the end of the three transformations, the point $(3, y)$ is on the resulting parabola.
   a. Find the equation of the image function after each transformation.
   b. Find the y-coordinate for the final graph, when $x = 3$.

Solution:

4. a. Reflection makes the image function $y = -x^2$.
   Vertical stretch makes the image function $y = -2x^2$.
   Translation makes the final image function $y = -2(x - 2)^2 - 4$.
   b. The final y-coordinate is –6.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1, by completing the square • determine p and q for question 3 • determine the image function after a single transformation in question 4.	• complete the square for question 2 • determine a for question 3 • find the final image function in question 4 • correctly determine the value of y for the point in question 4.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.3 Model real-world situations, using quadratic functions. [CN, PS]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Students must recognize that some real-world situations are described by quadratic functions.
- The solution of many real-world problems may involve rational expressions.
- Technology can be used to collect and analyze data.
- Tasks become routine for students when they have worked through many similar problems before.
- Novel tasks are those tasks that are unfamiliar to students.
- This specific outcome is related to specific outcome 3.6.
- Students are expected to present an algebraic expression to represent the problem.
- Students may solve these questions by completing the square, using the vertex formula or applying technology.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• work through routine questions that model real-world situations • work through novel real-world situations, given the functions that describe the situations • determine a solution but cannot analyze the solution for validity.	• work through routine and novel real-world situations, by determining the functions that describe the situations • analyze the validity of a solution.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.3 Model real-world situations, using quadratic functions. [CN, PS]

Sample Tasks

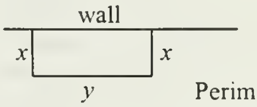
[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. What is the maximum rectangular area that can be enclosed by 120 m of fencing, if one of the sides of the rectangle is an existing wall?

Solution:

1.



Perimeter $= 2x + y = 120$

Area $= xy$

$= x(120 - 2x)$

$= -2x^2 + 120x$

$= -2(x^2 - 60x + 900) + 1800$

$= -2\,(x - 30)^2 + 1800$

Maximum area is 1800 m^2.

Question:

2. Computer software programs are sold to students for $20 each, and 300 students are willing to buy them at that price. For every $5 increase in price, there are 30 fewer students willing to buy the software. What is the maximum revenue?

Solution:

2. Let x be the number of $5 increases.

Revenue = Cost × number

$= (20 + 5x)\,(300 - 30x)$

$= -150x^2 + 900x + 6000$

$= -150\,(x^2 - 6x + 9) + 6000 + 1350$

$= -150\,(x - 3)^2 + 7350$

The maximum revenue is $7350.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.3 Model real-world situations, using quadratic functions. [CN, PS]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

3. The function $h(t) = -4.9t^2 + 5t + 2$ describes the height (h), in metres, of a ball at any time (t), in seconds.
   a. What is the height at $t = 1$ s?
   b. What is the maximum height the ball reaches?
   c. When does it reach its maximum height?
   d. Determine the time at which the ball reaches the height of 1.00 m.
   e. How long is the ball in the air?
   f. At $t = 0$, the height of the ball is 2 m. Explain.

Solution:

3. a. $h(t) = -4.9\,t^2 + 5t + 2$
   $h(1) = -4.9(1)^2 + 5(1) + 2$
   $h(1) = 2.1$
   The ball will be at a height of 2.1 m after 1 s.

   b. $h(t) = -4.9t^2 + 5t + 2$

   Vertex: $t = \dfrac{-b}{2a} = \dfrac{-5}{-9.8}$
   $= 0.51$ s

   $$h = \frac{4ac - b^2}{4a}$$

   $$= \frac{4(-4.9)(2) - 5^2}{4(-4.9)} = 3.28 \text{ m}$$

   The ball will reach a maximum height of 3.28 m.

   c. The ball reaches the maximum height after 0.51 s.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.3 Model real-world situations, using quadratic functions. [CN, PS]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

3. d. $h = -4.9\, t^2 + 5t + 2$

$1 = -4.9t^2 + 5t + 2$

$0 = -4.9t^2 + 5t + 1$

$$t = \frac{-5 \pm \sqrt{(5)^2 - 4(-4.9)(1)}}{2(-4.9)}$$

t = 1.19 s, as t must be positive.

The ball will reach a height of one metre in 1.19 seconds.

e. $h(t) = -4.9\, t^2 + 5t + 2$

$0 = -4.9t^2 + 5t + 2$

$$t = \frac{-5 \pm \sqrt{(5)^2 - 4(-4.9)(2)}}{2(-4.9)}$$

t = 1.33 s.

The ball is in the air for 1.33 seconds.

f. This represents the starting position, since when $t = 0$, $h(t) = 2$.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete question 1 • provide a partial solution to question 2 • complete parts a, b and c of question 3.	• determine an algebraic solution to question 2 • provide a complete solution to question 3.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
- factoring
- the quadratic formula
- graphing.

[CN, E, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Students are expected to solve quadratic equations by completing the square only when covering the derivation of the quadratic formula.
- The terminology associated with zeros, x-intercepts and roots is as follows: There are zeros of functions, x-intercepts of the graph of the function, and roots of the corresponding equation.
- Use technology to find zeros, x-intercepts and roots.
- For the Pure Mathematics 30 diploma examination, calculators must be cleared of any entered programs, including the quadratic formula.
- This specific outcome is related to specific outcome 2.1.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve basic quadratic equations by factoring • solve most quadratic equations using the quadratic formula, even when factoring may result in a faster solution • relate the x-intercepts of the graph of the function to the roots of the corresponding equation.	• solve quadratic equations, using any method; the student will choose factoring over the quadratic formula, if it leads to a faster solution • express irrational solutions in simplified mixed radical form.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
- factoring
- the quadratic formula
- graphing.

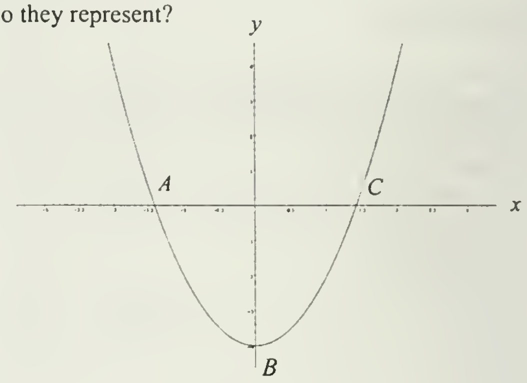
[CN, E, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. The graph of the quadratic function $y = 2x^2 - 4$ is given below. Three points, A, B and C, are shown. What are the values of A, B and C, and what do they represent?



Solution:

1. A is $(-\sqrt{2}, 0)$ and represents one solution of $2x^2 - 4 = 0$.
   B is $(0, -4)$ and represents the minimum value of $y = 2x^2 - 4$.
   C is $(\sqrt{2}, 0)$ and represents a second solution to $2x^2 - 4 = 0$.

Question:

2. Explain how the roots of $x^2 - 2x - 3 = 0$ can be obtained by graphing the functions $y = x^2$ *and* $y = 2x + 3$.

Solution:

2. The roots of $x^2 - 2x - 3 = 0$ will correspond to the points of intersection of $y = x^2$ and $y = 2x + 3$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
- factoring
- the quadratic formula
- graphing.

[CN, E, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

3. Write an integral quadratic function with roots $x = -2$ and $x = \frac{1}{3}$. Write the function in expanded form.

Solution:

3. $y = a(x + 2)(x - \frac{1}{3})$
Take $a = 3$
$y = (x + 2)(3x - 1)$
or $y = 3x^2 + 5x - 2$

Question:

4. Write a quadratic function in the form $y = a(x - r)(x - s)$ with x-intercepts –2 and 3 and a graph that passes through the point (–1, 6). Leave the answer in factored form.

Solution:

4. $y = a(x + 2)(x - 3)$
When $x = -1$, $y = 6$
$6 = a(-1 + 2)(-1 - 3)$
$a = -\frac{3}{2}$
$y = -\frac{3}{2}(x + 2)(x - 3)$

Question:

5. Use the quadratic formula to determine the exact values, in simplest form, for the solution set of $x^2 - 4x + 1 = 0$.

Solution:

5. $$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{12}}{2}$$
$$x = \frac{4 \pm \sqrt{4}\sqrt{3}}{2}$$
$$= \frac{2(2 \pm \sqrt{3})}{2}$$
$$x = 2 \pm \sqrt{3}$$

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
- factoring
- the quadratic formula
- graphing.

[CN, E, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

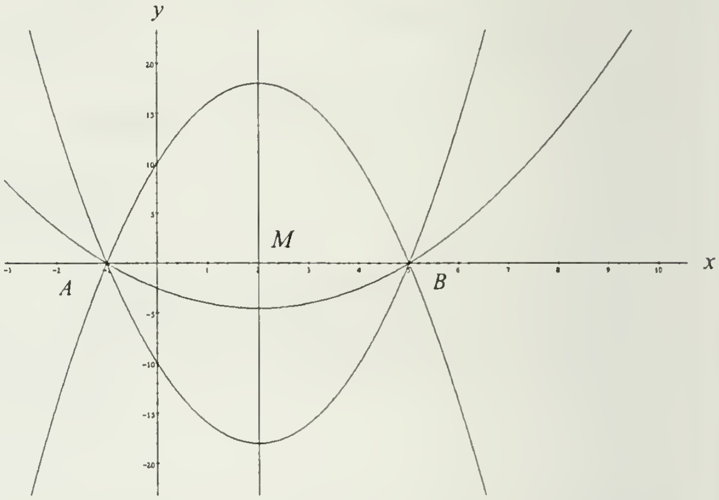
(continued)

Question:

6. The roots of a quadratic equation are –1 and 5. Where could the vertex of the corresponding quadratic function lie?
   a. Draw a sketch to illustrate where the vertex could lie.
   b. Explain where the vertex could be located.

Solution:

6. a.



b. The vertex $(2, y)$ lies anywhere on the line $x - 2 = 0$. The vertex lies on the axis of symmetry of the parabola. The axis of symmetry passes through the midpoint M of the line AB where $A = (-1, 0)$ and $B = (5, 0)$.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• identify the roots in question 1 • complete question 2 • determine the factors for question 3 • determine the function $y = (x + 2)(x - 3)$ in question 4 • determine the solution $x = \frac{4 \pm \sqrt{12}}{2}$ for question 5 • draw the graph of one parabola in question 6.	• describe what the points A, B and C represent in question 1 • determine the function in question 3 • determine the function $y = -\frac{3}{2}(x+2)(x-3)$ in question 4 • determine the solution $x = 2 \pm \sqrt{3}$ for question 5 • draw the graph of more than one parabola, and provide a clear explanation for part b in question 6.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.5 Determine the character of the real and non-real roots of a quadratic equation, using:
- the discriminant in the quadratic formula
- graphing.

[C, R, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Complex numbers are not part of the program of studies.
- The terminology for "character" or "nature" of roots differs in the approved resources.
- This specific outcome is related to specific outcome 3.6.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• calculate the discriminant for any quadratic equation • describe the nature of the roots • solve for parameters a, b or c in $b^2 - 4ac = 0$.	• solve for parameters a, b or c, given the nature of the roots.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.5 Determine the character of the real and non-real roots of a quadratic equation, using:
- the discriminant in the quadratic formula
- graphing.

[C, R, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Determine the value of the discriminant for $2x^2 + 3x - 7 = 0$, and state the nature of the roots.

Solution:

1. $b^2 - 4ac = 3^2 - 4(2)(-7)$
$= 9 + 56$
$= 65$
Since $b^2 - 4ac > 0$, there are two real roots.

Question:

2. a. Determine the value of k for the equation $x^2 + 4x + k = 0$, if the equation has real and equal roots.
b. What are the roots of the equation?

Solution:

2. a.
$$\begin{aligned} b^2 - 4ac &= 0 \\ 4^2 - 4(1)(k) &= 0 \\ 16 - 4k &= 0 \\ 4k &= 16 \\ k &= 4 \end{aligned}$$

b.
$$\begin{aligned} x^2 + 4x + 4 &= 0 \\ (x + 2)(x + 2) &= 0 \end{aligned}$$
Root is $x = -2$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.5 Determine the character of the real and non-real roots of a quadratic equation, using:
- the discriminant in the quadratic formula
- graphing.

[C, R, T, V]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

(continued)

Question:

3. For what values of k will the graph of $f(x)=x^2+4x+k$ cross the x-axis twice?

Solution:

3. $f(x) = x^2 + 4x + k$

$$\begin{aligned} b^2 - 4ac &> 0, \text{ if there are two crossing points} \\ 4^2 - 4(1)(k) &> 0 \\ 16 &> 4k \\ 4k &< 16 \\ k &< 4 \end{aligned}$$

Question:

4. For what values of k will the graph of $f(x) = 3x^2 + kx + 5$ not cross the x-axis?

Solution:

4. $f(x) = 3x^2 + kx + 5$

$$\begin{aligned} b^2 - 4ac &< 0 \\ k^2 - 4(3)(5) &< 0 \\ k^2 &< 60 \\ \therefore -7.75 < k &< 7.75 \end{aligned}$$

or

$$-2\sqrt{15} < k < 2\sqrt{15}$$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1 • determine the value of k in question 2 • solve question 3.	• determine the roots of the equation in question 2 • solve question 4.

STANDARDS IN POLYNOMIAL AND OTHER NONLINEAR EQUATIONS AND FUNCTIONS

GENERAL OUTCOMES

- Represent and analyze situations that involve expressions, equations and inequalities.
- Examine the nature of relations with an emphasis on functions.
- Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

SPECIFIC OUTCOMES

3.1 Solve nonlinear equations:
  - by factoring
  - graphically.

[CN, T, V]

3.2 Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]

3.3 Perform operations on functions and compositions of functions. [CN, E, PS]

3.4 Determine the inverse of a function. [CN, R, V]

3.5 Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]

3.6 Formulate and apply strategies to solve:
  - absolute value equations
  - radical equations
  - rational equations
  - quadratic inequalities
  - polynomial inequalities.

[CN, R, T, V]

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.1 Solve nonlinear equations:
- by factoring
- graphically.

[CN, T, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- Factoring involves polynomial equations only and should be covered concurrently with specific outcome 3.2.
- Repeated factors of the form $(x - a)^2$, $(ax - b)^2$, and $(x - a)^3$ are included as part of this outcome.
- Other nonlinear equations are covered in specific outcomes 3.5 and 3.6.
- Students should be able to provide the following information in the window of the calculator.

```
Y1 =
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =
```

```
WINDOW
  Xmin=
  Xmax=
  Xscl=
  Ymin=
  Ymax=
  Yscl=
```

- It is appropriate to include questions that require the use of the quadratic formula (specific outcome 2.5) for a complete solution.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve polynomial equations of degree three or more but will have difficulty solving questions that require using the Remainder Theorem and Factor Theorem unless guidance is provided • solve nonlinear equations graphically but will tend to use one method only, either setting one side equal to zero or finding points of intersection between two graphs.	• solve polynomial equations of degree three or more and is able to use the Remainder Theorem and Factor Theorem effectively when necessary • solve nonlinear equations graphically and is flexible in using different methods, depending on which is more appropriate.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.1 Solve nonlinear equations:
- by factoring
- graphically.

[CN, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Solve by factoring:
$2x^3 + 3x^2 - 8x - 12 = 0$

Solution:

1. $2x^3 + 3x^2 - 8x - 12 = 0$

$$x^2(2x+3) - 4(2x+3) = 0$$
$$(x^2 - 4)(2x+3) = 0$$
$$(x-2)(x+2)(2x+3) = 0$$
$$x = 2, -2, -\frac{3}{2}$$

Question:

2. Solve by factoring:
$x^3 - 2x + 1 = 0$

Solution:

2. $P(x) = x^3 - 2x + 1$

$P(1) = 1 - 2 + 1 = 0$

$$\begin{array}{rl}
 & x^2 + x - 1 \\
(x-1) & \overline{\big)\, x^3 + 0x^2 - 2x + 1} \\
 & \underline{x^3 - x^2 \phantom{{}-2x+1}} \\
 & \phantom{x^3 - {}} x^2 - 2x \\
 & \underline{\phantom{x^3 - {}} x^2 - x } \\
 & \phantom{x^3 - x^2 - {}} -x + 1 \\
 & \underline{\phantom{x^3 - x^2 - {}} -x + 1} \\
 & \phantom{x^3 - x^2 - -x + {}} 0
\end{array}$$

Factors: $(x-1)(x^2 + x - 1)$.

Using the quadratic formula for $x^2 + x - 1 = 0$:

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2} \text{ and } x = 1.$$

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.1 Solve nonlinear equations:
- by factoring
- graphically.

[CN, T, V]

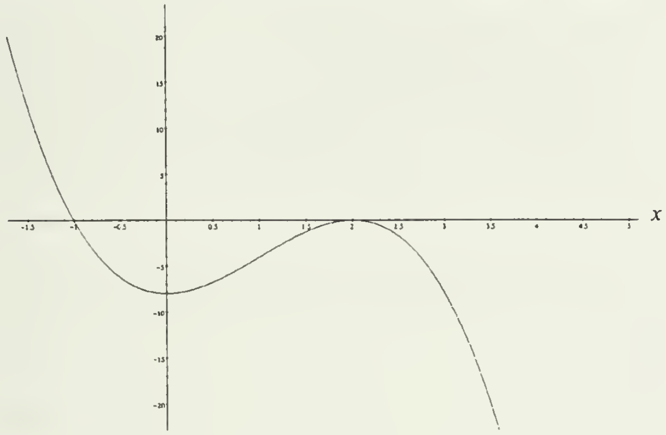
Sample Tasks

☐ Conceptual
☐ Procedural
☑ Problem-solving

(continued)

Question:

3. A partial graph of a cubic function is shown. Determine the equation in factored form.



Solution:

3. $y = a(x + 1)(x - 2)(x - 2)$ with $y(0) = -8$
   $-8 = 4a$
   $y = -2(x + 1)(x - 2)(x - 2)$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1 • determine one root for question 2 • verify $(x + 1)$ and $(x - 2)$ as factors in question 3.	• determine all the roots for question 2 • provide a complete solution for question 3.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Complex numbers are not part of the program of studies.
- The Integral and Rational Zero Theorems are not required parts of the specific outcome.
- The application of the Remainder and Factor Theorems should be limited to integral zeros.
- Repeated factors of the form $(x-a)^2$, $(ax-b)^2$, and $(x-a)^3$ are included as part of this outcome.
- Integral zeros must be used to reduce polynomial equations to second degree form.
- Systems of equations, specific outcome 1.2, should be covered before this specific outcome.
- Synthetic division is not a required part of the specific outcome, but many teachers may include it as a timesaving method of division by factors of form $(x-a)$.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine if $(x-a)$ is a factor of a polynomial • determine the remainder, if a polynomial is divided by $x-a$ • determine one coefficient in a polynomial, given the factors.	• apply the Remainder and Factor Theorems to solve for more than one coefficient in a polynomial.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Factor $3x^3 + x^2 - 8x + 4$.

Solution:

1. $P(-2) = 3(-2)^3 + (-2)^2 - 8(-2) + 4 = 0$
   $(x + 2)$ is a factor
   $(3x^2 - 5x + 2)$ is the other factor
   $\therefore (x - 1)\,(x + 2)\,(3x - 2)$ are the factors.

Question:

2. Given the graph $y = P(x)$, determine the remainder when $P(x)$ is divided by $x - 2$.

Solution:

2. Remainder $= P(2) = -12$

Question:

3. The polynomial $P(x) = 3x^3 + kx^2 + 3x - 5$ has a remainder of 9 when divided by $x - 2$. Determine the value of k.

Solution:

3. $P(2) = 9;\ P(2) = 3(2)^3 + k(2)^2 + 3(2) - 5$

$$\begin{aligned} 9 &= 24 + 4k + 6 - 5 \\ 9 &= 25 + 4k \\ -16 &= 4k \\ \therefore k &= -4 \end{aligned}$$

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Question:

4. The polynomial $P(x) = 4x^3 + bx^2 - cx + 11$ has a remainder of -7 when divided by $x + 2$ and a remainder of 14 when divided by $x - 1$. Determine the values of b and c.

Solution:

4. $P(-2) = 7$

$-7 = 4(-2)^3 + b(-2)^2 - c(-2) + 11$

$-7 = -32 + 4b + 2c + 11$

$P(1) = 14$

$14 = 4(1)^3 + b(1) - c(1) + 11$
$14 = 4b + 2c$

$14 = 4 + b - c + 11$
$-1 = b - c$

Solve:
$14 = 4b + 2c$
$-1 = b - c$
$b = 2$; $c = 3$.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete questions 1, 2 and 3.	• complete question 4.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.3 Perform operations on functions and compositions of functions. [CN, E, PS]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The approved resources use different notations for compositions of functions. Students should be familiar with the notation $f(g(x))$ and $(f \bullet g)(x)$ for the composition of f and g.
- Although this specific outcome does not refer to technology or visualization, if time permits it is worthwhile to make connections between the graphs of the original functions and the graphs of the functions resulting from the operations and/or compositions.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• perform operations on functions and compositions of functions.	• recognize that domain and range will change for the new function, and identify the new domain and range.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.3 Perform operations on functions and compositions of functions. [CN, E, PS]

Sample Task

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. If $f(x)=x+2$ and $g(x)=x^2-4$, then determine the following and state any restrictions on the variable.

   a. $f(2)+g(3)$

   b. $f(g(4))$

   c. $f(x)+g(x)$

   d. $f(x)-g(x)$

   e. $f(x)\bullet g(x)$

   f. $\dfrac{f(x)}{g(x)}$

   g. $f(g(x))$

   h. $g(f(x))$

Solution:

1. a. $f(2)+g(3)=4+5$
   $\quad\quad\quad\quad\quad=9$

   b. $f(g(4))$
   $g(4)=4^2-4=12$
   $f(12)=12+2=14$

   c. $f(x)+g(x)=(x+2)+(x^2-4)$
   $\quad\quad\quad\quad\quad=x^2+x-2$

   d. $f(x)-g(x)=(x+2)-(x^2-4)$
   $\quad\quad\quad\quad\quad=-x^2+x+6$

   e. $f(x)\bullet g(x)=(x+2)(x^2-4)$
   $\quad\quad\quad\quad\quad=x^3+2x^2-4x-8$

(continued)

General and Specific Outcomes
General Outcome Examine the nature of relations with an emphasis on functions. **Specific Outcome** 3.3 Perform operations on functions and compositions of functions. [CN, E, PS]

Sample Task

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution:

1. f. $\dfrac{f(x)}{g(x)} = \dfrac{x+2}{x^2-4} = \dfrac{x+2}{(x+2)(x-2)} = \dfrac{1}{x-2}$

$x \neq \pm 2$

g. $f(g(x)) = (x^2-4)+2$
$= x^2 - 2$

h. $g(f(x)) = (x+2)^2 - 4$
$= x^2 + 4x + 4 - 4$
$= x^2 + 4x$

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete parts a through e • develop partial solutions to parts f through h.	• complete the solutions to parts f through h, and state the restrictions.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.4 Determine the inverse of a function. [CN, R, V]

Notes:

- In McGraw-Hill Ryerson *Mathpower 11*, the $f^{-1}(x)$ notation is used for the inverse of any function, regardless of whether or not the inverse is a function.
- In Addison-Wesley *Mathematics 11*, $f^{-1}(x)$ notation is used only if the inverse is a function, otherwise it is written in the $y =$ form.
- When students are writing equations for inverse functions, restrictions on the domain and range of the inverse should be noted; e.g., if $f(x) = 1 - \sqrt{x}$, then $f^{-1}(x) = x^2 - 2x + 1$, and $f^{-1}(x) \geq 0$, $x \leq 1$.
- Technology is a useful tool for drawing the graphs of inverse functions.
- The teaching of reciprocal functions as a separate topic is not part of this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• recognize $f^{-1}(x)$ notation • determine the inverse of a function.	• differentiate $f^{-1}(x)$ notation from $\frac{1}{f(x)}$ • determine if $f^{-1}(x)$ is a function • determine the domain and range of the inverse function.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.4 Determine the inverse of a function. [CN, R, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. Write the inverse of $f(x) = x^2 + 2$; state if the inverse is a function; and state its domain and range, if applicable.

Solution:

1. Inverse: $x = y^2 + 2$ or $f^{-1}(x) = \pm\sqrt{x - 2}$. The inverse is not a function. The domain is $x \geq 2$, and the range is $y \in R$.

Question:

2. Samantha incorrectly determined the inverse of $y = 1 - \sqrt{x}$ to be $y = x^2 - 2x + 1$, and she used a graphing tool to sketch the inverse. Explain why the graph of the correct inverse is not a parabola.

Solution:

2. The domain and range of $y = 1 - \sqrt{x}$ are $x \geq 1$ and $y \leq 1$. Therefore, the domain and range of the inverse must be $x \leq 1$ and $y \geq 1$. When Samantha determined the inverse, she should have restricted the domain as follows:

$x = 1 - \sqrt{y}, x \leq 1$

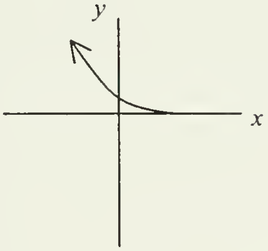
$\sqrt{y} = 1 - x, x \leq 1$

$(\sqrt{y})^2 = (1 - x)^2, x \leq 1$

$y = (1 - x)^2, x \leq 1$

$y = x^2 - 2x + 1, x \leq 1$

The graph is only the left half of a parabola.



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine the inverse and the domain and range in question 1, including one square root only • sketch the inverse of the function in question 2.	• determine the inverse, including both square roots and the domain and range, in question 1 • include the restrictions on the domain and range in question 2 • explain why the inverse in question 2 is not a complete parabola.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.5 Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- **This specific outcome can be a time trap.**
- Students should be familiar with the terms "continuous" and "discontinuous" in terms of functions.
- In describing polynomial functions, the following characteristics should be considered:
  - identify that a graph represents a polynomial function
  - leading coefficient
  - degree
  - shape
  - maximum or minimum values, using technology
  - domain and range—graphically only for range
  - x- and y-intercepts
  - multiplicity of roots—terminology in basic resources may vary.
- Double root means the root has a multiplicity of 2.
- In describing rational functions the following characteristics should be considered:
  - identify that a graph represents a rational function
  - shape
  - vertical and horizontal asymptotes—no other asymptotes
  - domain and range—graphically only for range
  - maximum and minimum values—terminology in basic resources may vary
  - x- and y-intercepts.
- The teaching of reciprocal functions as a separate topic is not part of this specific outcome.
- Students can be expected to solve for the roots of a polynomial equation. Integral roots can be found algebraically, and nonintegral roots can be found graphically.
- Students should be able to relate the zeros of the polynomial function to the solution of the corresponding polynomial equation.
- Students should be able to relate the zeros of a rational function $\frac{N(x)}{D(x)}$ to the zeros of $N(x)$.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• manipulate the window of the calculator to determine some of the characteristics of polynomial and rational functions • find most characteristics of polynomial and rational functions • determine the domain, including where there are vertical asymptotes • identify the horizontal and vertical asymptotes.	• identify all characteristics of both polynomial and rational functions, using appropriate terminology • determine the equations of the horizontal and vertical asymptotes.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.5 Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]

Sample Tasks

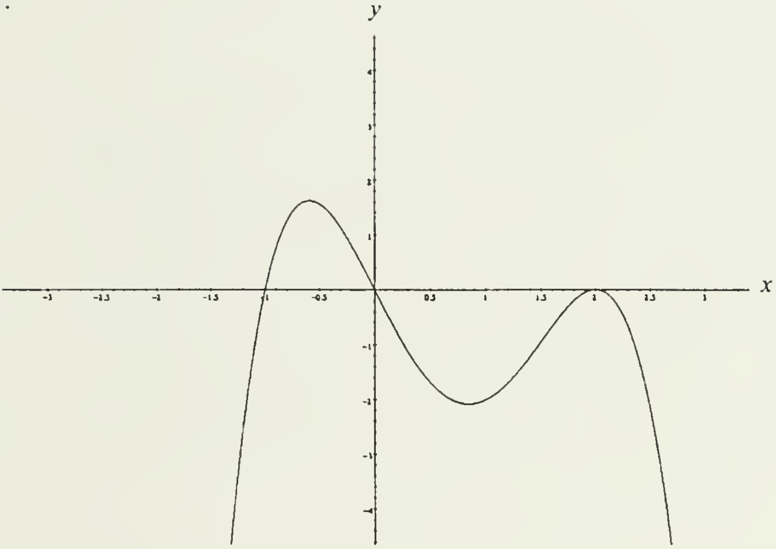
☑ Conceptual
☐ Procedural
☐ Problem-solving

Question:

1. Using technology, graph $P(x) = -x^4 + 3x^3 - 4x$.
   a. State the x-intercepts.
   b. State the multiplicity of the roots of $P(x) = 0$.
   c. Determine the maximum and minimum values to one decimal place.
   d. State the domain and range, correct to one decimal place.
   e. State the range when $x < -1$.
   f. Express $P(x)$ in factored form.

Solution:

1.



a. 2, 0, –1
b. The root 2 is a double root—multiplicity of 2.
   The roots 0 and –1 are each single roots—multiplicity of 1.
c. local maxima 1.6 and 0
   local minimum –2.1
d. $x \in R \quad y \leq 1.6$
e. $y < 0$
f. $P(x) = -x(x-2)^2(x+1)$

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.5 Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

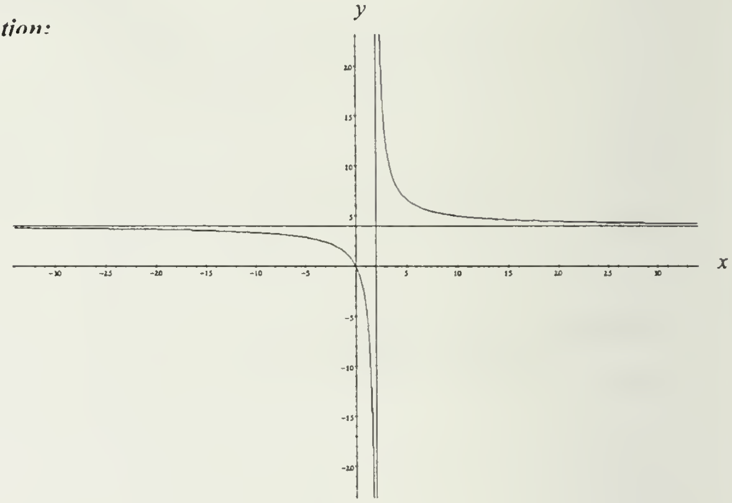
Question:

2. Use technology to graph $y = \dfrac{4x}{x-2}$.

   a. State the vertical asymptote.
   b. State the horizontal asymptote.
   c. State the domain and range.
   d. State the *x*- and *y*-intercepts.

Solution:

2.



   a. $x = 2$
   b. $y = 4$
   c. domain $x \neq 2$; range $y \neq 4$
   d. x-intercept = 0
      y-intercept = 0

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete question 1, parts a, b and c; state the domain; and identify the factors of the polynomial function • identify the vertical asymptote and the domain in question 2.	• complete all parts of questions 1 and 2.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- **This specific outcome can be a time trap.**
- This specific outcome includes:
  - absolute value equations
  - radical equations
  - rational equations and rational inequalities.
- Absolute value and radical inequalities are excluded from this specific outcome.
- Algebraic solutions should be restricted to:
  - single absolute value equations, such as $|2x-1|=3$
  - radical equations, such as $\sqrt{x}=2x-6$ or $\sqrt{x}+\sqrt{x+3}=3$
  - the simple rational equations, such as $\frac{x^2-5x-6}{x+1}=2$.
- All other equations and inequalities should be solved graphically.
- Students should be exposed to both algebraic and technological means of solving these equations and inequalities.
- Graphical solutions should include identification of window settings, function(s) entered, and sketch(es) of the displayed graph(s).
- The solution to inequalities should be written algebraically; e.g., $x \leq 2$.
- Solving equations with more than two absolute value expressions is beyond the scope of this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve equations and inequalities both algebraically and by using technology, but will not necessarily restrict the solution based on the domain and range of the original function(s) • solve radical equations where the radicals can be eliminated in one step • solve absolute value equations involving just one absolute value expression • solve very simple polynomial inequalities of the form $N(x) > k$, where k is a real number • solve two-term rational equations.	• solve equations and inequalities both algebraically and by using technology, and will restrict the solution based on the domain and range of the original function(s), if necessary • solve radical equations that require more than one step to eliminate the radical • solve absolute value equations with two absolute value expressions • solve more complex polynomial inequalities, such as those with more than two terms.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

Sample Tasks

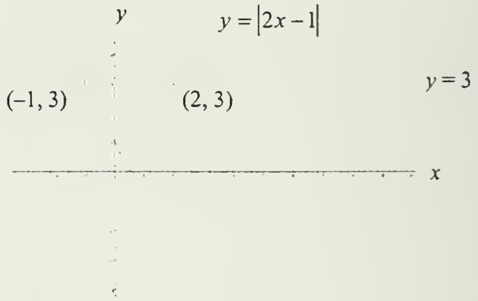
[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. Solve the following.
   a. $|2x-1|=3$, graphically and algebraically
   b. $|x|+|x+2|=5$, graphically
   c. $\sqrt{2x+1}=x-1$, graphically and algebraically
   d. $\sqrt{x}+\sqrt{x+5}=5$, graphically and algebraically (optional)

Solution:

1. a. Graphical solution: Graph $y=|2x-1|$ and $y=3$, and determine the points of intersection.



The points of intersection are (–1, 3) and (2, 3); therefore, the solutions are –1 and 2.

An alternative graphical solution is to graph $y=|2x-1|-3$ and find its zeros.

Algebraic solution:

Case 1: $x \geq \frac{1}{2}$	Case 2: $x < \frac{1}{2}$
$2x-1=3$	$-(2x-1)=3$
$2x=4$	$2x-1=-3$
$x=2$	$2x=-2$
	$x=-1$

Therefore, $x=-1$ or $x=2$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

Sample Tasks

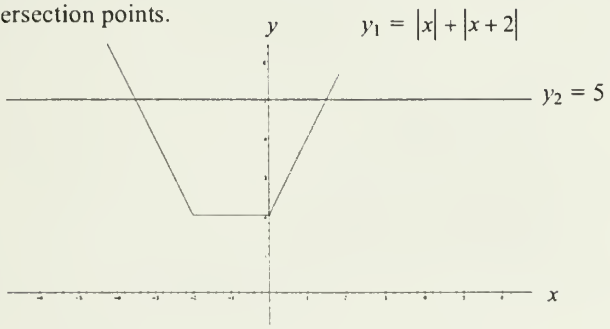
[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

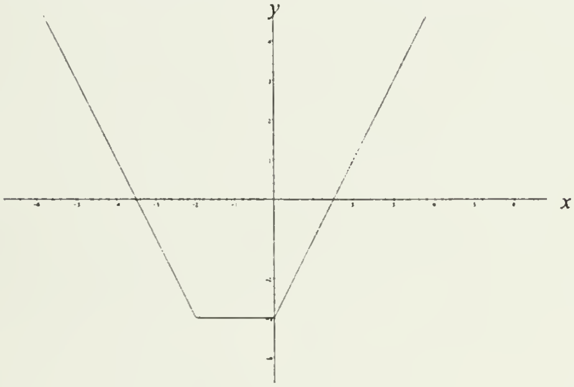
1. b. Graphical solutions:

Solution 1: Graph $y_1 = |x| + |x+2|$ and $y_2 = 5$, and find the intersection points.



Intersection points have x-coordinates of 1.5 and –3.5.

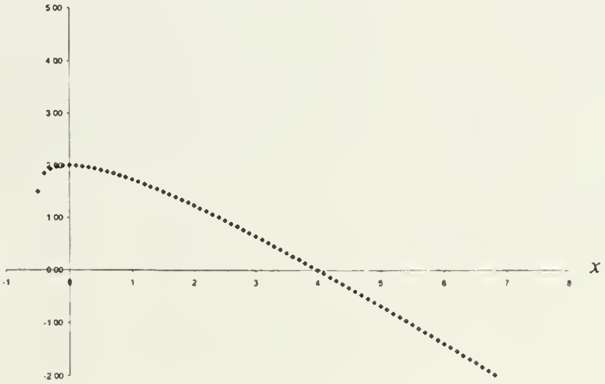
Solution 2: Graph $y = |x| + |x+2| - 5$, and find its zeros.



Zeros are –3.5 and 1.5.

c. Graphical solutions:

Solution 1: Find the zeros of $y = \sqrt{2x+1} - (x-1)$.



The zero is at $x = 4$. *(continued)*

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

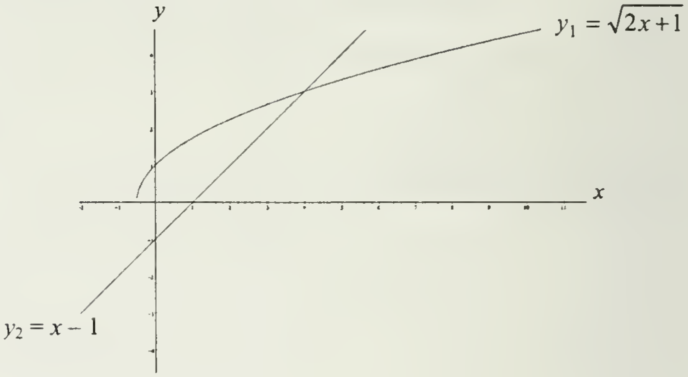
Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

1. c. Solution 2: Find the intersection point of $y_1 = \sqrt{2x+1}$ and $y_2 = x - 1$. This point is (4, 3), so $x = 4$ is the solution of the original equation.



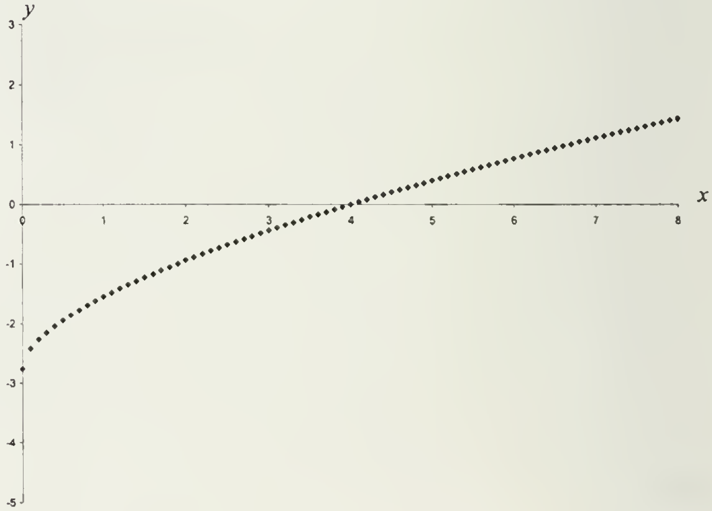
Algebraic solution:

$$\sqrt{2x+1} = x - 1$$
$$2x + 1 = (x - 1)^2$$
$$0 = x^2 - 4x$$
$$0 = x(x - 4)$$

So $x = 0$ or $x = 4$.
Check: $x = 0$ gives a left side of 1 and a right side of -1, so is not a solution. $x = 4$ is a solution as both sides are equal to 3.
So $x = 4$ is the only solution.

d. Graphical solutions:

Solution 1: Graph $y = \sqrt{x} + \sqrt{x+5} - 5$, and find its zeros.



The zero is at $x = 4$.

(continued)

General and Specific Outcomes
General Outcome Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate. **Specific Outcome** 3.6 Formulate and apply strategies to solve: • absolute value equations • radical equations • rational equations • quadratic inequalities • polynomial inequalities. [CN, R, T, V]

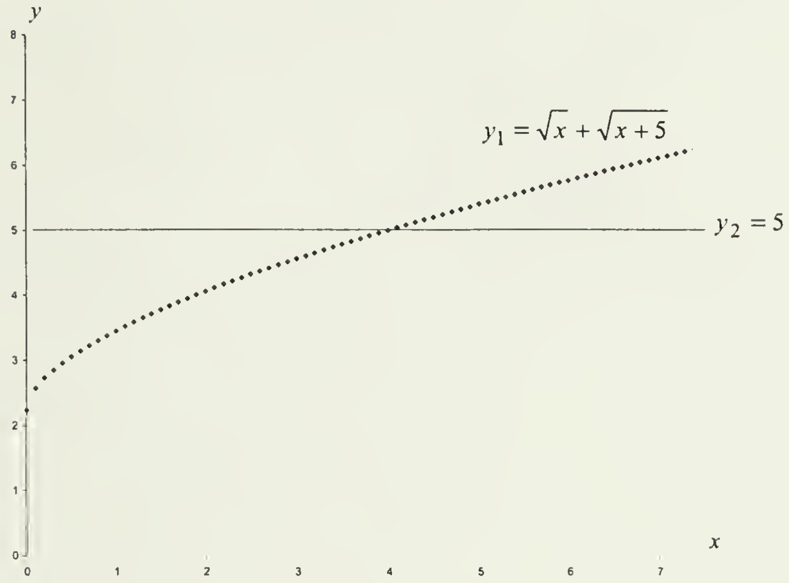
Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

1. d. Solution 2: Graph $y_1 = \sqrt{x} + \sqrt{x+5}$ and $y_2 = 5$, and find the x-coordinate of the point of intersection.



The x-coordinate of the point of intersection is 4.

Algebraic solution (optional):

$$\begin{aligned}\sqrt{x} + \sqrt{x+5} &= 5\\ \sqrt{x+5} &= 5 - \sqrt{x}\\ x+5 &= \left(5-\sqrt{x}\right)^2\\ x+5 &= 25 - 10\sqrt{x} + x\\ 10\sqrt{x} &= 20\\ \sqrt{x} &= 2\\ x &= 4\end{aligned}$$

Check: $\sqrt{x} + \sqrt{x+5} = \sqrt{4} + \sqrt{9} = 2 + 3 = 5$

left side = right side.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

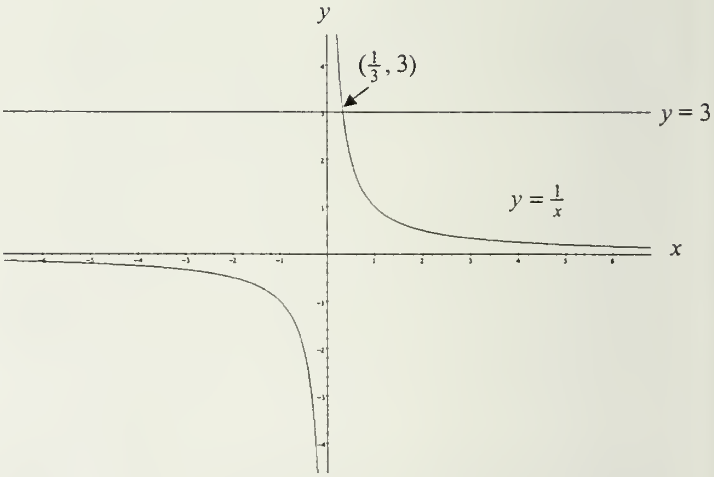
2. Solve the following graphically.

a. $\dfrac{1}{x} > 3$

b. $\dfrac{6}{x+3} > x + 8$

Solution:

2. a.



So $0 < x < \frac{1}{3}$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

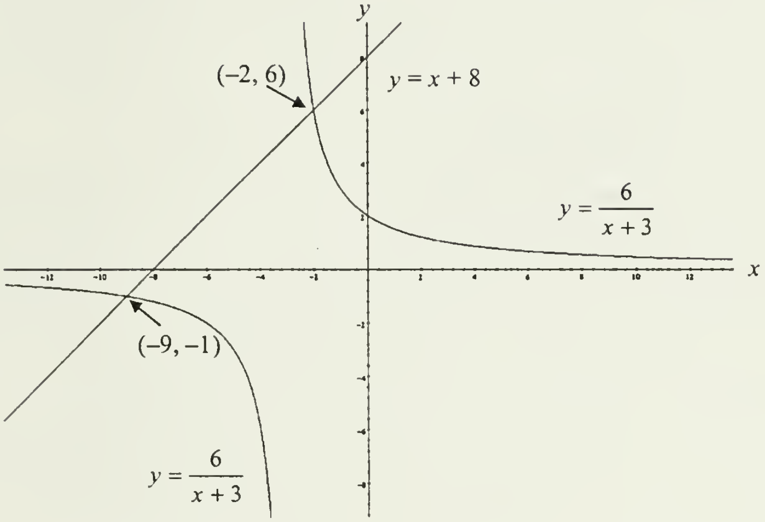
Sample Tasks

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

2. b. Graphical solution. Graph $y = x + 8$ and $y = \frac{6}{x+3}$.



So $x < -9$, or $-3 < x < -2$.

Question:

3. A railway company increases the speed of its high-speed trains by 120 km/h over a 540 km journey. The time saved is 72 minutes. Calculate the original and new speeds for the trains.

Solution:

3. Let the original speed of the train be v km/h, so the new speed will be $(v + 120)$ km/h.

As time is distance/speed, and the time difference is 1.2 h, the following equation applies:

$$\frac{540}{v} - \frac{540}{v+120} = 1.2$$

$$540(v + 120) - 540v = 1.2v(v + 120)$$
$$64\,800 = 1.2v^2 + 144v$$
$$v^2 + 120v - 54\,000 = 0$$
$$(v + 300)(v - 180) = 0$$

So the original speed of the train was 180 km/h, and the new speed is 300 km/h.

Check: Original time = (540 km)/(180 km/h) = 3 h
New time = (540 km)/(300 km/h) = 1.8 h
Time saved = 1.2 h, as required.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Question:

4. The royalties received by an author depend on the number sold and on the price paid by the consumer. For a particular book, the royalties $R(p)$ depend on the price p according to the equation $R(p) = 0.02p(20\,000 - 200p)$. For what range of prices does the author receive more than \$8400 in royalties?

Solution:

4. $R(p) = 0.02p(20\,000 - 200p)$ must be greater than \$8400.

So $0.02p(20\,000 - 200p) > 8400$
$-4p^2 + 400p - 8400 > 0$
$(-4p + 120)(p - 70) > 0$

So either $(-4p + 120)$ and $(p - 70)$ are both positive, or they are both negative.

They cannot both be positive, as then p would have to be less than 30 and more than 70 at the same time.

They can both be negative, if p is between 30 and 70.

Therefore, the range of selling prices is between \$30 and \$70 per book.

As an alternative to inspecting the signs of the expressions $(-4p + 120)$ and $(p - 70)$, the parabolic graph of $R(p) = 4p^2 + 400p - 8400$ can be sketched, and the range of prices is the interval between the zeros of $R(p)$. Many students find the graphical approach easier than the use of intervals.

Question:

5. An open box is constructed from a 36 cm by 24 cm sheet of cardboard by cutting out squares of side x cm from each corner and then folding up the sides. The volume of such a box is given by the cubic function $V(x) = x(24 - 2x)(36 - 2x)$.

a. For what values of x is $V(x) > 0$?
b. Do all these values of x represent real boxes?

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

3.6 Formulate and apply strategies to solve:
- absolute value equations
- radical equations
- rational equations
- quadratic inequalities
- polynomial inequalities.

[CN, R, T, V]

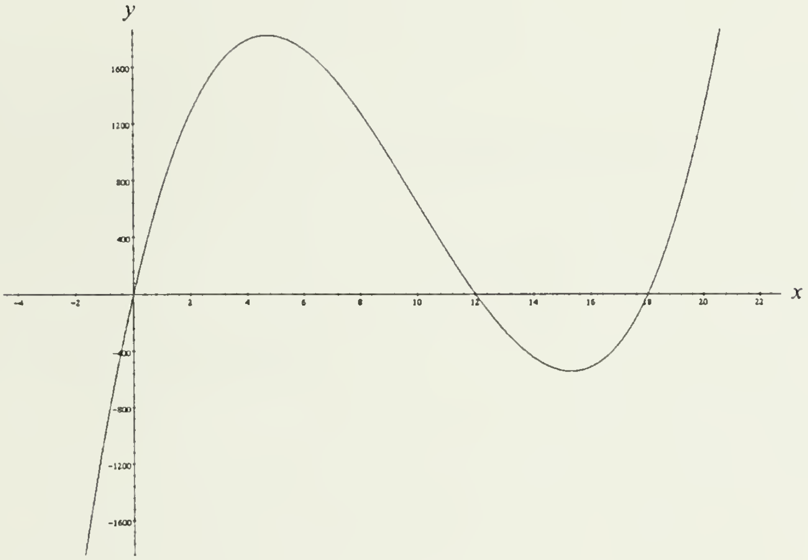
Sample Tasks

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

5. a. The graph of $V(x)$ is shown below, and if $0 < x < 12$, or if $x > 18$, then $V(x) > 0$.



b. If $0 < x < 12$, then each of the three factors of $V(x) = x(24 - 2x)(36 - 2x)$ are positive numbers, so can represent the length, width and height of a real box. However, if $x > 18$, then the factors $(24 - 2x)$ and $(36 - 2x)$ are both negative, and so cannot represent the length and width of a real box.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide a complete solution to question 1, part a, and a graph for question 1, part b • solve question 1, part c, graphically • solve question 2, part a, as $x < \frac{1}{3}$ • draw the graphs for question 2, part b • solve question 3 • give an example of an acceptable price in question 4 • solve question 5, part a.	• provide a complete solution to question 1, part b • provide a complete solution to question 1, part c, noting the extraneous root • solve question 1, part d, either algebraically or graphically • solve question 2, part a, completely • solve question 2, part b, completely • solve question 4 completely • solve question 5, part b.

STANDARDS IN FORMAL REASONING

GENERAL OUTCOME

- Apply the principles of mathematical reasoning to solve problems and to justify solutions.

SPECIFIC OUTCOMES

4.1 Differentiate between inductive and deductive reasoning. [CN, R]

4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]

4.3 Use examples and counterexamples to analyze conjectures. [CN, R]

4.4 Distinguish between an "if–then" proposition, its converse and its contrapositive. [CN, R]

4.5 Prove assertions in a variety of settings, using direct reasoning. [R]

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.1 Differentiate between inductive and deductive reasoning. [CN, R]

Notes:

- Specific outcomes 4.1 to 4.5 follow from geometry concepts studied in junior high mathematics: Grade 7—properties of parallel lines, transformations; Grade 8—classifying of polygons; Grade 9—transformation properties, locus definitions, similar and congruent triangles.
- Students entering Pure Mathematics 20 should have the background knowledge to make conjectures but not necessarily to prove them.
- Examples should be chosen from the real world, as well as from mathematical concepts related to algebra and geometry.
- Students should be made aware that inductive reasoning leads to conjectures, which may not always be true.
- Students should be encouraged to compile a list of geometric and algebraic facts.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• describe the difference between inductive and deductive reasoning • classify a given proof as inductive or deductive • produce an inductive proof that leads to a true conclusion.	• produce an inductive proof that leads to a false conclusion.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.1 Differentiate between inductive and deductive reasoning. [CN, R]

Sample Tasks

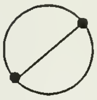
[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. Draw diagrams to show how many distinct regions are formed when:
   a. 2 points on a circle are joined
   b. 3 points on a circle are joined
   c. 5 points are placed on a circle and all possible chords are drawn.

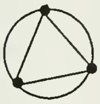
Solution:

1. a.



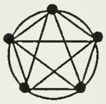
2 regions

b.



4 regions

c.



16 regions

Question:

2. Use what you learned in question 1 to write a formula that gives the number of regions formed when n points are placed on a circle and all possible chords are drawn. Is the formula true or false?

Solution:

2. Number of regions = 2^{n-1}, where n = the number of points on a circle for $n > 2$. This formula is validated for $2 \le n \le 5$, but remains a conjecture for $n > 5$. This formula is false for $n = 6$.

Question:

3. Use deductive reasoning to show that the difference of squares of two odd numbers is divisible by 4.

(continued)

General and Specific Outcomes
General Outcome Apply the principles of mathematical reasoning to solve problems and to justify solutions. **Specific Outcome** 4.1 Differentiate between inductive and deductive reasoning. [CN, R]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

3. Difference of squares $= (2n + 1)^2 - (2m + 1)^2$

$$= 4n^2 + 4n + 1 - 4m^2 - 4m - 1$$
$$= 4[n^2 + n - m^2 - m]$$

If $n \neq m$, $n^2 + n - m^2 - m$ is an integer and so the difference of squares is a multiple of 4.

If $n = m$, $n^2 + n - m^2 - m = 0$ and the difference of squares is zero, which is a multiple of 4.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete question 1 • write a formula for question 2 • write an expression for the difference of squares for question 3.	• complete question 2, including the counterexample • complete question 3, including both cases.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Examples should be chosen from the real world, as well as from mathematical concepts in algebra and geometry.
- Students should understand that more conditions limit the chance of success and more options increase the chance of success.
- Applications of connecting words, such as *and*, *or* and *not*, can be found in the advanced search features of Internet search engines, in Venn diagrams and in computer programming.
- Particular care is needed when using the word *or*, as it can be used inclusively—either A or B or both—or exclusively—either A or B but not both—in ordinary speech.
- Topics covered in this specific outcome link strongly to the concept of conditional probability found in Pure Mathematics 30.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• illustrate the connecting words *and* and *or*, using number lines or Venn diagrams—limit to two circles • illustrate the connecting words *and* and *or*, using inequalities • draw and interpret a variety of two-circle Venn diagrams • identify the inclusive and exclusive use of the connecting word *or* in a real-life example • interpret a given three-circle Venn diagram.	• draw a three-circle Venn diagram for a set of data • illustrate the connecting word *not*, using inequalities and/or Venn diagrams • explain the difference between the inclusive and exclusive use of the connecting word *or*.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]

Sample Tasks

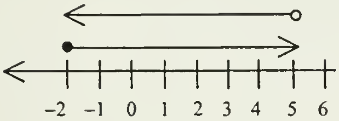
[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. Illustrate on a number line the following statements:
   a. $x < 5$ and $x \geq -2$
   b. $x < 5$ or $x > 8$

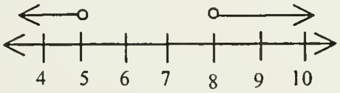
Solution:

1. a.



$\{-2 \leq x < 5\}$

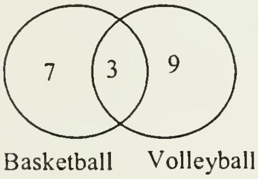
b.



$\{x < 5 \text{ or } x > 8\}$

Question:

2. The Venn diagram displays the number of boys who play on basketball and volleyball teams.



a. How may boys play basketball and not volleyball?
b. How many boys play both basketball and volleyball?

Solution:

2. a. Basketball only : 7 boys
   b. Both basketball and volleyball : 3 boys

(continued)

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

(continued)

Question:

3. Students were asked about the language courses they took in high school. The results of the survey and the total enrollment in each course were as follows.

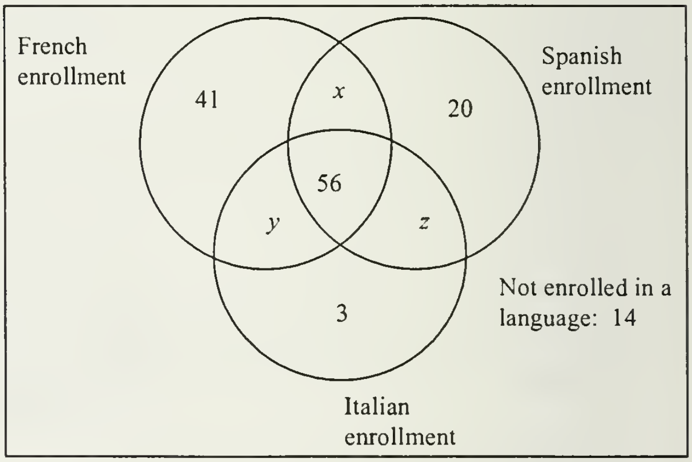
Language Courses Taken	Number of Students
French only	41
Spanish only	20
Italian only	3
All three: French, Spanish and Italian	56
Not taking a language	14

Total enrollment in French: 131 students
Total enrollment in Spanish: 115 students
Total enrollment in Italian: 82 students

a. Draw a Venn diagram to summarize this data.
b. The survey did not ask students if they were taking two of the three language courses. Determine these three unknown values.
c. How many students were surveyed?

Solution:

3. a.



b. Let x be the number of students enrolled in both French and Spanish.
Let y be the number of students enrolled in both French and Italian.
Let z be the number of students enrolled in both Spanish and Italian.

(continued)

General and Specific Outcomes
General Outcome Apply the principles of mathematical reasoning to solve problems and to justify solutions. **Specific Outcome** 4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

(continued)

Solution:

3. b. Then
French enrollment = $x + y + 41 + 56 = 131$
Spanish enrollment = $x + z + 20 + 56 = 115$
Italian enrollment = $y + z + 3 + 56 = 82$

So:
$x + y = 34$
$x + z = 39$
$y + z = 23$

And:
$y - z = -5$
$y + z = 23$

Therefore:
$x = 25$
$y = 9$
$z = 14$

c. Total surveyed:

$41 + 56 + 20 + 3 + 14 + 25 + 9 + 14 = 182$ students.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1 • complete question 2 • draw the Venn diagram for question 3.	• provide a complete solution to question 3.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.3 Use examples and counterexamples to analyze conjectures. [CN, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Students entering Pure Mathematics 20 should have the background knowledge to make conjectures but not necessarily to prove them.
- Examples should be chosen from the real world, as well as from mathematical concepts in algebra and geometry.
- Consider situations where counterexamples are obvious and those where counterexamples are difficult to find.
- Distinguish between conjectures that are reasonable and those that can be proved to be true.
- Graphing calculator screens provide a rich source of algebraic conjectures.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• define conjecture • make conjectures and predictions based on patterns in a set of data • provide an obvious counterexample to a conjecture, based on a real-life experience • provide a counterexample to a conjecture, based on a simple mathematical concept.	• provide a counterexample to a conjecture, based on a mathematical concept where the counterexample is not obvious • recognize that the absence of a counterexample does not constitute proof of a conjecture.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.3 Use examples and counterexamples to analyze conjectures. [CN, R]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. Give an example and a counterexample for the following.
If $a < b$, then $k \times a < k \times b$.

Solution:

1. example: $a = 3, b = 4, k = 2$
$3 < 4$
then $2 \times 3 < 2 \times 4$
$6 < 8$, which is true.

counterexample: $a = 3, b = 4, k = -2$
$3 < 4$
$-2 \times 3 < -2 \times 4$
$-6 < -8$, which is false.

Question:

2. When Mary graphed $f(x) = x^x$ for $x < 0$, she observed a blank screen. She made the conjecture that x^x is undefined for $x < 0$. Give an example and a counterexample for this conjecture.

Solution:

2. example: $x = -\frac{1}{2}$

$$\left(-\frac{1}{2}\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{-0.5}}$$

which is undefined. So the conjecture is true for this example.

counterexample: $x = -2$

$$(-2)^{-2} = \frac{1}{(-2)^2} = \frac{1}{4}$$

So the conjecture is false.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• provide both an example and a counterexample for question 1 • provide an example for question 2.	• provide a counterexample for question 2.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.4 Distinguish between an "if–then" proposition, its converse and its contrapositive. [CN, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This is intended as an introductory topic and, thus, should be kept simple.
- Detailed treatments, using such techniques as truth tables, are beyond the scope of this specific outcome.
- Students should examine "if–then" statements that are initially false, as well as those that are initially true.
- Examples should be chosen from the real world, as well as from mathematical concepts in algebra and geometry.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• write the converse and contrapositive of a given statement in "if–then" form, using both real life and mathematical situations • give an example of an "if–then" statement where: – the converse is true – the converse is false.	• give an example of an "if–then" statement where the contrapositive is true • give an example of an "if–then" statement where the contrapositive is false • make links between the truth of the original statement and the truth of the contrapositive and converse • analyze converses and contrapositives for "if–then" statements that are initially false.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.4 Distinguish between an "if–then" proposition, its converse and its contrapositive. [CN, R]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. "If it rains, then I will go to the mall."
   a. Write the converse and contrapositive of this true conditional statement.
   b. If the conditional statement is true, are the converse and contrapositive necessarily true?

Solution:

1. a. Converse: "If I go to the mall, then it is raining."
   Contrapositive: "If I don't go to the mall, then it is not raining."
   b. The converse is not necessarily true, while the contrapositive is true.

Question:

2. "If a number is odd, then it is a prime number."
   a. Write the converse and contrapositive of this conditional statement.
   b. Is this conditional statement true or false?
   c. Are the converse and contrapositive true or false? Justify your answers.

Solution:

2. a. Converse: "If a number is prime, it is odd."
   Contrapositive: "If a number is not odd, then it is not prime."
   b. False. Many odd numbers are not prime numbers. Such a number could be 15, 21 or many more.
   c. The converse is false as 2 is even but is prime. The contrapositive is false — 2 is both even and prime.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• write a converse statement, given the true conditional statement in question 1 • judge the truth of statements in real-life situations, starting from a true conditional statement, such as that in question 1.	• write a contrapositive statement for question 1 • write and judge the truth of a converse statement and a contrapositive statement, given the false conditional statement in question 2.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.5 Prove assertions in a variety of settings, using direct reasoning. [R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Proofs should be based on a mixture of algebraic, geometric and trigonometric approaches; and proofs may start with theorems; e.g., Pythagorean theorem.
- The methods of direct proof can be introduced here and embedded throughout specific outcomes 5.2 to 5.5.
- Students should be allowed to use a variety of formats to present their proofs.
- Students should not be expected to provide more than one alternative format for any given proof.
- Students need to communicate clearly their starting points, their reasons and their conclusions.
- Formal names for theorems used are not required.
- Some students may define figures in terms of their transformational properties and justify congruence in terms of symmetry lines, translations or rotations.
- Examples should be chosen from the real world, as well as from mathematical concepts in algebra and geometry.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide reasons for statements in proofs, using direct reasoning • complete a simple proof, using direct reasoning, once the first two or three steps are given.	• construct a short proof, requiring up to five lines, using direct reasoning • complete a more complex proof, using direct reasoning, once the first steps are given.

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.5 Prove assertions in a variety of settings, using direct reasoning. [R]

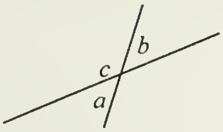
Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. When lines intersect, opposite angles are equal. Prove this statement.

Solution:



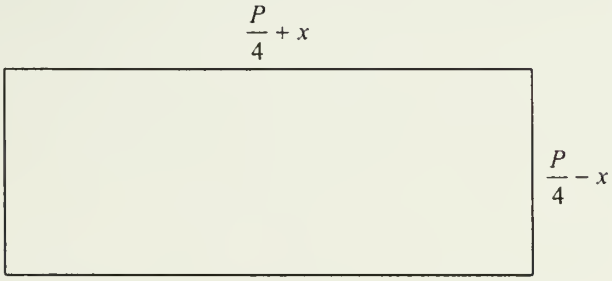
1. $\angle a + \angle c = 180°$ (straight line)
   $\angle b + \angle c = 180°$ (straight line)
   $\angle a + \angle c = \angle b + \angle c$
   $\quad - \angle c \qquad - \angle c$
   $\therefore \angle a = \angle b$

Question:

2. Prove that of all rectangles with a given perimeter, the square has the largest area.

Solution:

2. Proof (narrative style):



For any rectangle of perimeter P, the length can be expressed as $\frac{P}{4} + x$, and the width as $\frac{P}{4} - x$. The area is the product, or $\left(\frac{P}{4} + x\right)\left(\frac{P}{4} - x\right)$, which is $\frac{P^2}{16} - x^2$. The largest area is whenever x is zero, as $\frac{P^2}{16} - x^2$ is always less than or equal to $\frac{P^2}{16}$.

When x is zero, the length and the width are equal, and the rectangle is a square.

(continued)

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.5 Prove assertions in a variety of settings, using direct reasoning. [R]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

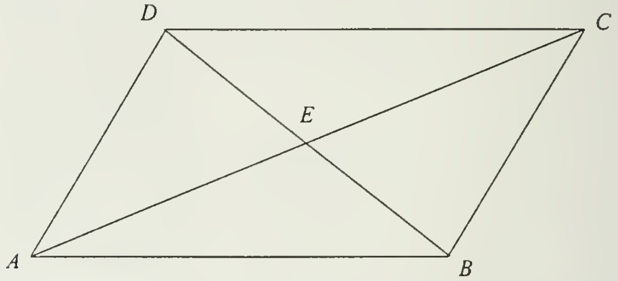
(continued)

Question:

3. Prove that the diagonals of a parallelogram bisect one another.

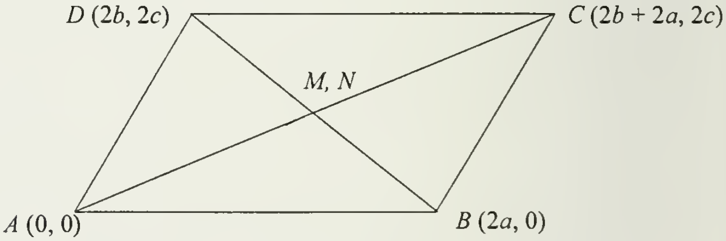
Solution:

3. Proof 1 (two-column format):



Statement	Reason
1. $AB = CD$	Opposite sides of a parallelogram are equal in length.
2. $m(\text{angle } CDE) = m(\text{angle } ABE)$	Alternate angles created by a transversal to parallel lines AB and DC are equal.
3. $m(\text{angle } DCE) = m(\text{angle } BAE)$	Alternate angles created by a transversal to parallel lines AB and DC are equal.
4. Triangles DCE, BAE congruent	Angle-side-angle.
5. $DE = BE$, and $AE = CE$	Corresponding sides of congruent triangles are equal.
6. Diagonals of a parallelogram bisect one another.	Restatement of $DE = BE$, and $AE = CE$.

Proof 2 (coordinate geometry):



Choose axes and an origin so that $A = (0, 0)$ and $B = (2a, 0)$.
Choose D as $(2b, 2c)$.
As CD is equal in length to AB, and parallel to it, CD is horizontal and of length $2a$.
So C is $(2b + 2a, 2c)$.
Then the midpoint M of AC is $(b + a, c)$, and the midpoint N of BD is:
$\left(\frac{1}{2}(2a+2b)\right), \left(\frac{1}{2}(0+2c)\right)$ or $(a + b, c)$.

Since M and N have the same coordinates, they represent the same point. Therefore, the diagonals of the parallelogram bisect one another.

(continued)

General and Specific Outcomes

General Outcome

Apply the principles of mathematical reasoning to solve problems and to justify solutions.

Specific Outcome

4.5 Prove assertions in a variety of settings, using direct reasoning. [R]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

4. Prove that any even, three-digit number, whose digits add to a multiple of 3, is itself divisible by 6.

Solution:

4. Proof:

The three-digit, even number N can be represented by the digits pqr, where $r = 2s$, because N is known to be even.

The digits p, q and r are related to the value of N by the expression $N = 100p + 10q + \mathrm{r}$.

The digit sum $p + q + r = k$, where k is a whole number.

The digit p can, therefore, be replaced by the expression $3k - q - r$, or $3k - q - 2s$.

So $N = 100p + 10q + r$, or $N = 100(3k - q - 2s) + 10q + 2s$.

Now collecting like terms in the expression N:

$N = 300k - 90q - 198s$, or $N = 6(50k - 15q - 33s)$. Therefore, N is always divisible by 6.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1, given the diagram • complete question 2, if the first sentence of the proof is given • provide reasons for the first proof in question 3 • complete the second proof in question 3, given the diagram • verify one or two specific examples in question 4.	• complete question 1, constructing his/her own diagram • complete questions 2 and 3, including the provision of relevant diagrams • complete question 4 for the general case.

STANDARDS IN CIRCLES AND COORDINATE GEOMETRY

GENERAL OUTCOMES

- Develop and apply the geometric properties of circles and polygons to solve problems.
- Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

SPECIFIC OUTCOMES

5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles.

  [PS, R, T, V]

5.2 Prove the following general properties, using established concepts and theorems:
  - the perpendicular bisector of a chord contains the centre of the circle
  - the angle inscribed in a semicircle is a right angle
  - the tangent segments to a circle from any external point are congruent.

  [PS, R, V]

5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used. [PS, R, V]

5.4 Solve problems involving distances between points and lines. [CN, PS, R]

5.5 Verify and prove assertions in plane geometry, using coordinate geometry and trigonometric ratios as necessary. [C, R, V]

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- the angle inscribed in a semicircle is a right angle
- the opposite angles of a cyclic quadrilateral are supplementary
- a tangent to a circle is perpendicular to the radius at the point of tangency
- the tangent segments to a circle, from any external point, are congruent
- the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
- the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles.

[PS, R, T, V]

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 5.1, except for the eighth bullet related to the angle between a tangent and a chord.
- This specific outcome follows from geometry concepts studied in junior high mathematics: Grade 7 — properties of parallel lines, transformations; Grade 8 — classifying of polygons; Grade 9 — transformation properties, locus definitions, similar and congruent triangles.
- The use of geometry software with animation, such as *Geometer's Sketchpad* or *Cabri*, can help produce the large number of special cases needed to establish conjectures by inductive reasoning. Many examples do not require software.
- Drafting programs without animation do not save any time compared to drawing the figures by hand.
- Examples should be chosen from the real world, as well as from mathematical concepts in algebra and geometry.
- *Geometer's Sketchpad* only measures angles up to 180°.
- The Pythagorean Theorem and the trigonometry of right triangles are useful in this specific outcome.
- To save time, instruction in this specific outcome can be integrated with specific outcomes 5.2 and 5.3.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine the measure of an angle or segment when the use of one property of circles or polygons is required • determine the measures of angles or segments when the use of more than one property of circles or polygons is required and the diagram is given.	• determine the measures of angles or segments when the use of more than one property of circles or polygons is required and the diagram is not given • determine the measures of angles or segments when the use of other geometric properties, such as parallel lines and congruent triangles, as well as the properties of circles or polygons are required.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- the angle inscribed in a semicircle is a right angle
- the opposite angles of a cyclic quadrilateral are supplementary
- a tangent to a circle is perpendicular to the radius at the point of tangency
- the tangent segments to a circle, from any external point, are congruent
- the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
- the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles.

[PS, R, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

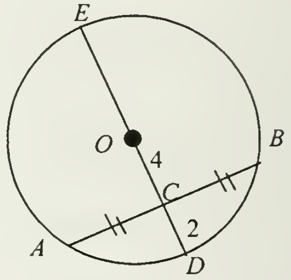
1. Use animated geometry software to investigate the measure of the inscribed angle in a semicircle.

Solution:

1. Solutions will vary; however, the measure is always 90°.

Question:

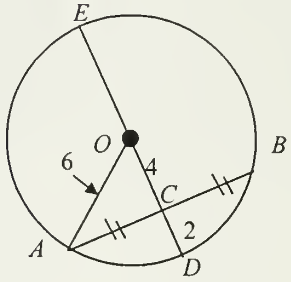
2. Given the diagram below, determine:
   a. $\overline{AC}$
   b. $\overline{CE}$



$r = 6$

Solution:

2.



a. $\overline{AC} = \sqrt{6^2 - 4^2}$
$= \sqrt{36 - 16}$
$= \sqrt{20}$

b. $\overline{CE} = 6 + 4$
$= 10$

(continued)

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles.

  [PS, R, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

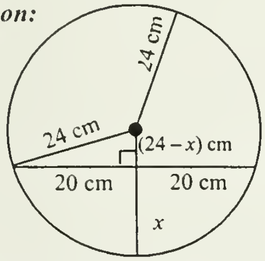
(continued)

Question:

3. A circular water pipe has a radius of 24 cm. If the width of the water surface in the pipe is 40 cm, find the maximum depth of the water to the nearest tenth of a centimetre.

Solution:

3.



Water Pipe Cross Section

$r = 24$ cm
width of water surface = 40 cm
water depth; let it be x cm

$$(24 - x)^2 + (20)^2 = (24)^2$$
$$576 - 48x + x^2 + 400 = 576$$
$$x^2 - 48x + 400 = 0$$
$$x = \frac{48 \pm \sqrt{(48)^2 - 4(1)(400)}}{2}$$
$$x \doteq 37.27 \text{ cm or } 10.73 \text{ cm}$$

The depth of the water is approximately 37.3 cm or 10.7 cm.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• perform the investigation in question 1 with guided instruction • complete question 2, given the additional information that $\overline{OA} = 6$.	• perform question 1 independently • complete question 2 • find one solution for question 3, if a diagram is not given • find both solutions for question 3, if a diagram is given.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Prove the following general properties, using established concepts and theorems:
- the perpendicular bisector of a chord contains the centre of the circle
- the angle inscribed in a semicircle is a right angle
- the tangent segments to a circle from any external point are congruent.

[PS, R, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The intent of this specific outcome is to assist students in understanding the nature of proof and to assist students in their reading of proofs presented in textbooks.
- Students should be exposed to a variety of formats for presenting proofs.
- Formal names for theorems used are not required.
- Figures may be defined in terms of their transformational properties, symmetry lines, rotations or translations.
- Students should be expected to paraphrase teacher-presented proofs.
- The understanding of the structure of a proof gained here, as well as the understanding of the concepts and theorems, can be assessed by asking students to fill in gaps and paraphrase.
- Proofs for each of these properties can be found in the student textbooks.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• demonstrate understanding of the problem by providing a verification for one or more special cases, not a proof • provide reasons for statements of proofs described in this specific outcome • start the proof of a related conjecture.	• demonstrate understanding of the problem by providing a proof for the general case • provide reasons for the steps in a complete proof of a related conjecture.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Prove the following general properties, using established concepts and theorems:
- the perpendicular bisector of a chord contains the centre of the circle
- the angle inscribed in a semicircle is a right angle
- the tangent segments to a circle from any external point are congruent.
  [PS, R, V]

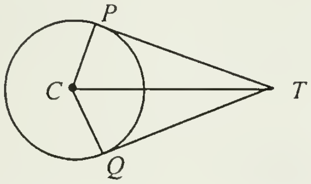
Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Tangents *TP* and *TQ* are drawn from an external point *T* to a circle, with centre *C*. Prove that quadrilateral *TPCQ* is cyclic.

Solution **(one of many possible):**



1. Angle *TPC* is equal to 90°, as the line segment *TP* is a tangent, and the line segment *CP* is a radius.
   Angle *TQC* is equal to 90°, as the line segment *TQ* is a tangent, and the line segment *CQ* is a radius.
   So the sum of the opposite angles *TPC* and *TQC* is 180°.
   Therefore, quadrilateral *TPCQ* is cyclic, as opposite angles have a sum equal to 180°.

(continued)

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Prove the following general properties, using established concepts and theorems:
- the perpendicular bisector of a chord contains the centre of the circle
- the angle inscribed in a semicircle is a right angle
- the tangent segments to a circle from any external point are congruent.

[PS, R, V]

Sample Tasks

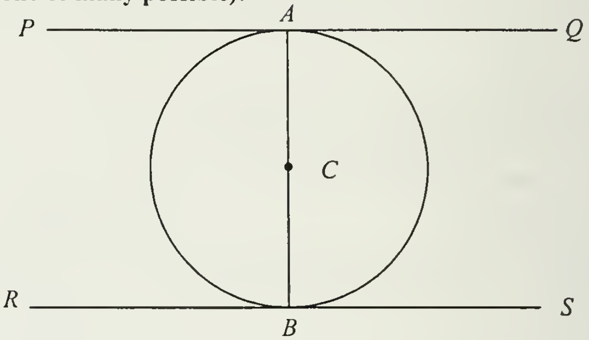
[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

2. Prove that the two tangents at the ends of a diameter to a circle are parallel to each other.

Solution **(one of many possible):**



2. In the diagram above, the tangents are PAQ and RBS, with ACB being the diameter.

   Angle PAC is equal to 90°, as PA is a tangent and AC is a radius.

   Also, angle SBC is equal to 90°, as SB is a tangent and BC is a radius.

   But angles PAC and SBC are alternate angles; so then PAQ and RBS are parallel, as their alternate angles are equal.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• provide reasons for the steps in the proof in question 1, after the diagram and the steps have been given • paraphrase the proof in question 2.	• complete the proofs of both questions, with the diagram given for question 1 and not given for question 2.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used.
[PS, R, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Problems should be limited to those that require the use of the circle properties identified in specific outcomes 5.1 and 5.2.
- This specific outcome may require the use of either trigonometry or coordinate geometry.
- The specific outcome may involve verification of special cases but does not involve proofs.
- Problems involving arc length and sector area are optional here, and these can be covered in the Pure Mathematics 30 specific outcome on degree and radian measure.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide solutions to simple problems, and explain the solution strategies used • solve complex problems.	• provide complete solutions to complex problems, and explain the solution strategies used • justify the solution strategy used to solve a problem.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

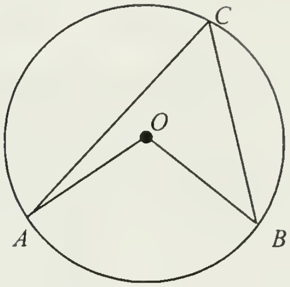
5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used. [PS, R, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1.



If chord $AB = 11.7$ cm and radius $OA = 10.0$ cm, find the angle ACB, where C lies on the major arc AB. State the circle properties used in the solution.

Solution:

1. Join AB; midpoint is M.

$$\text{Angle } AOM = \sin^{-1}\left[\frac{AM}{OA}\right]$$
$$= \sin^{-1}\left[\frac{5.85}{10.0}\right] = 35.8°$$
$$\text{Angle } AOB = 2 \times \text{angle } AOM$$
$$= (2)\,(35.8°)$$
$$= 71.6°$$
$$\angle ACB = \text{half } \angle AOB$$
$$= 35.8°$$

Circle properties used:

- OM is perpendicular to AB, if M is the midpoint of AB.
- OM bisects $\angle AOB$.
- $\angle ACB$ is half $\angle AOB$.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used.
[PS, R, V]

Sample Tasks

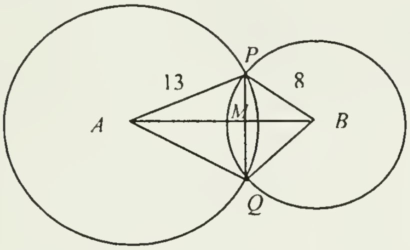
☐ Conceptual
☐ Procedural
☑ Problem-solving

(continued)

Question:

2. Two circles have radii of 13 cm and 8 cm. Their centres are 18 cm apart. Find the length of their common chord to the nearest 0.1 cm. Explain solution strategy used.

Solution:



Apply the cosine law to triangle *APB*.

$$\cos \angle PAB = \left[\frac{13^2 + 18^2 - 8^2}{2(13)(18)}\right]$$

$\angle PAB = 23.56°$

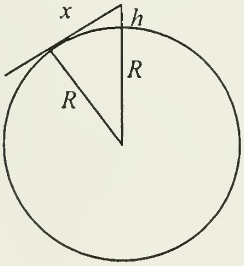
As *AB* is the perpendicular bisector of *PQ*, triangle *PMA* is right angled.

$$\begin{aligned} PQ &= 2\,(13 \sin \angle PAB) \\ &= 10.39 \text{ cm} \\ &= 10.4 \text{ cm} \end{aligned}$$

Question:

3. Jake raises his eye height from 1.55 m to 1.70 m with platform shoes. How much further can he see? Assume the radius of the Earth is 6370 km. Explain the strategy used.

Solution:



without shoes
$x^2 + R^2 = (R + h)^2$
$x^2 + 6\,370\,000^2 = 6\,370\,001.55^2$
$x = 4443.76$ m

with shoes, $h = 1.70$ m
$x^2 + 6\,370\,000^2 = 6\,370\,001.70^2$
$x = 4653.82$ m

He can see (4653.8 – 4443.8) or 210 m further.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used.
[PS, R, V]

Sample Tasks

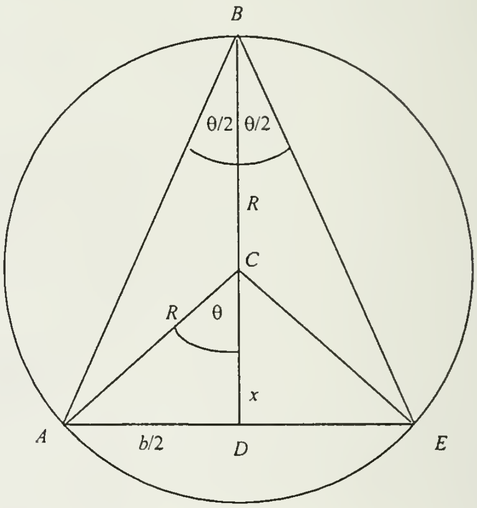
[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

4. An isosceles triangle is inscribed in a circle of radius R. Derive expressions for the area of the triangle in terms of:
   a. the radius and the length of the base
   b. the radius and the vertex angle.

Solution:



4. a. $A = \frac{1}{2}bh$

Now substitute $h = R + x$, so $A = \frac{1}{2}b(R + x)$.

Use Pythagorean theorem to eliminate x: $x = \sqrt{R^2 - \left(\frac{b}{2}\right)^2}$

Substitute for x to obtain the final expression:

$$A = \frac{1}{2}b\left(R + \sqrt{R^2 - \frac{b^2}{4}}\right)$$

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

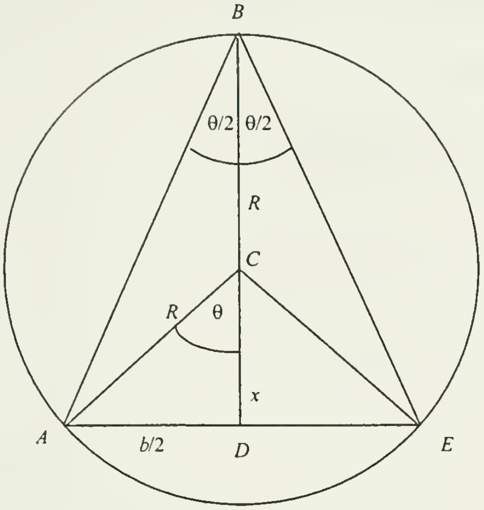
5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used.
[PS, R, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:



4. b. $A = \frac{1}{2}bh$

Now substitute $h = R + x$, so $A = \frac{1}{2}b(R + x)$.

Vertex angle ABE is half of central angle ACE, so angle $ABD = \frac{1}{2}\theta$, and angle $ACD = \theta$.

From the right triangle ACD, we have $x = R\cos\theta$ and $b = 2R\sin\theta$.

Substitute for x and b to obtain the final expression:

$$A = \frac{1}{2}\left(2R\sin\theta\right)\left(R + R\cos\theta\right)$$

This simplifies to $A = R^2 \sin\theta(1 + \cos\theta)$.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1, if the diagram is given • provide limited explanations of the solution strategies in question 1 • start question 3, if the diagram is given • solve question 4, part a, if the diagram is given.	• complete question 1, if the diagram is not given • provide detailed explanations of the solution strategies in question 1 • complete question 2, if the diagram is given • draw the diagram in question 3 • complete question 3 • solve question 4, part a, if the diagram is not given • solve question 4, part b, if the diagram is given.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.4 Solve problems involving distances between points and lines. [CN, PS, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Distinctions should be made among the vertical distance from a point to a line, the horizontal distance from a point to a line and the shortest distance from a point to a line.
- The equation of the circle, as derived from the distance formula, can be a useful tool to solve problems related to this specific outcome and in the construction of coordinate proofs.
- The horizontal distance is optional, depending on how the shortest distance is calculated.
- The general formula for the shortest distance is not required.
- Either trigonometric or systems methods can be used for the shortest distance.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• illustrate vertical distance from a point to a line • determine the vertical distance from a point to a line expressed in the $y = mx + b$ form • represent, graphically, the shortest distance from a point to a line given in the form $y = mx + b$ • begin an algebraic solution to determine the shortest distance from a point to a line • determine the vertical distance between two parallel lines given in the form $y = mx + b$.	• determine the vertical distance from a point to a line given in the form $Ax + By + C = 0$ • represent, graphically, the shortest distance from a point to a line given in the form $Ax + By + C = 0$ • determine, algebraically, the shortest distance from a point to a line • describe the procedure needed to determine the shortest distance between two parallel lines.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

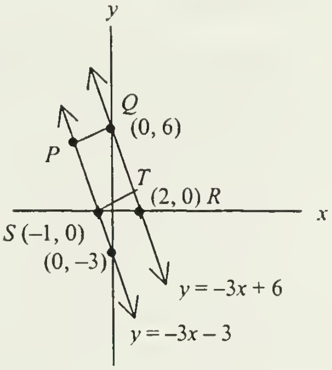
5.4 Solve problems involving distances between points and lines.
[CN, PS, R]

Sample Task

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Find the vertical distance and the shortest distance between the parallel lines $y = -3x - 3$ and $y = -3x + 6$.



Solution 1:

Vertical Distance:
Find the y-intercepts

$y = -3x - 3$	$y = -3x + 6$
$y = -3$	$y = 6$

Vertical distance = 9.

Shortest Distance:
Any line perpendicular to $y = -3x + 6$ has a slope of $\frac{1}{3}$.

① $m = \frac{1}{3}$
passes through (0, 6)
$y - y_1 = m(x - x_1)$
$y - 6 = \frac{1}{3}(x - 0)$
$y = \frac{1}{3}x + 6$ is the equation of PQ.

② Shortest Distance:

$$y = \frac{1}{3}x + 6 \text{ ①}$$
$$-\ \underline{(y = -3x - 3)\text{ ②}}$$
$$0 = \frac{10}{3}x + 9$$
$$-9 = \frac{10}{3}x$$
$$-\frac{27}{10} = x$$
$$y = -3\left(-\frac{27}{10}\right) - 3$$
$$y = \frac{51}{10}$$

The coordinates of P are $\left(-\frac{27}{10}, \frac{51}{10}\right)$

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.4 Solve problems involving distances between points and lines. [CN, PS, R]

Sample Task

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution 1:

③ Use the distance formula for points P, Q.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-\tfrac{27}{10} - 0)^2 + (\tfrac{51}{10} - 6)^2} \\ &= \sqrt{\tfrac{729}{100} + \tfrac{81}{100}} \\ &= \sqrt{\tfrac{810}{100}} \\ d &\doteq 2.85 \end{aligned}$$

The shortest distance PQ is estimated to be 2.85.

Solution 2:

Horizontal Distance:
Find the x-intercepts

$$\begin{array}{ll} y = -3x - 3 & y = -3x + 6 \\ 0 = -3x - 3 & 0 = -3x + 6 \\ -1 = x & 2 = x \end{array}$$

Horizontal distance = 3.

Shortest Distance:

$$\begin{aligned} &ST = SR \sin(\angle SRT) \\ &\text{Now } \tan(\angle SRT) = 3 \\ &\therefore \angle SRT = 71.57^\circ \\ &\quad ST = 3 \sin 71.57^\circ \\ &\qquad = 2.85 \end{aligned}$$

The shortest distance ST is approximately 2.85.

Descriptions of Student Performance (Related to Sample Task)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• find the vertical distance • find the horizontal distance, if using the method of solution 2.	• find the vertical or horizontal distance, and use it to find the shortest distance.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.5 Verify and prove assertions in plane geometry, using coordinate geometry and trigonometric ratios as necessary. [C, R, V]

Notes:

- Students may use a variety of formats to present their proofs.
- Students need to communicate clearly their starting points, their reasons and their conclusions.
- Formal names for theorems used are not required.
- The midpoint, slope and distance formulas from Pure Mathematics 10 are essential to this specific outcome.
- Some students may define figures in terms of their transformational properties and justify congruence in terms of symmetry lines, translations or rotations.
- By placing a key point in the problem at the origin and orienting the figure along either the x- or y-axis, it is possible to minimize the algebra involved in coordinate geometry proofs. If this is not done, the algebra is usually unmanageable.
- The equation of the circle, as derived from the distance formula, can be a useful tool to solve problems related to this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• verify conjectures, given the coordinates of specific points, using coordinate geometry • differentiate between verification for special cases and proof for general cases • complete a short proof of a geometric property, using coordinate geometry, when a labelled diagram is provided.	• provide the diagram and complete a two- to four-line proof of a geometric property, using coordinate geometry • complete a longer proof of a geometric property, using coordinate geometry, when a labelled diagram is provided • select and explain appropriate positions for the origin and orientations of the coordinate axes.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.5 Verify and prove assertions in plane geometry, using coordinate geometry and trigonometric ratios as necessary. [C, R, V]

Sample Tasks

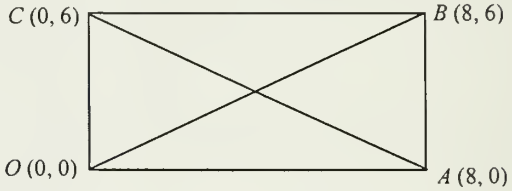
[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A rectangle *OABC* has *O* at the point (0, 0), *A* at (8, 0) and *C* at the point (0, 6). Find the coordinates of the point *B*, and verify that the diagonals *OB* and *AC* are equal in length.

Solution:

1.



$$\begin{aligned} OB^2 &= OA^2 + AB^2 \\ &= 8^2 + 6^2 \\ &= 100 \end{aligned}$$

$$\begin{aligned} AC^2 &= OA^2 + OC^2 \\ &= 8^2 + 6^2 \\ &= 100 \end{aligned}$$

$\therefore\ OB = AC = 10$

Question:

2. A line segment has end points *M*(8, 6) and *N*(–6, 8).
   a. Show that *MN* is a chord of the circle $x^2 + y^2 = 100$.
   b. Determine the equation of the line that passes through the centre of the circle and the midpoint of chord *MN*.
   c. Explain how you can tell from your answer in part b that the line is perpendicular to the chord *MN*.

Solution:

2. a. left side

$M: 8^2 + (6)^2$
$64 + 36$
$100 =$ right side
$N: (-6)^2 + 8^2$
$36 + 64$
$100 =$ right side

Both *M* and *N* satisfy $x^2 + y^2 = 100$, so they lie on the circle $x^2 + y^2 = 100$.
Therefore, *MN* is a chord of the circle.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.5 Verify and prove assertions in plane geometry, using coordinate geometry and trigonometric ratios as necessary. [C, R, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

2. b. $\text{Mid}_{MN} = \left(\frac{8 + -6}{2}, \frac{6+8}{2}\right)$

$= (1, 7)$

$C = (0, 0)$ is the centre of the circle $x^2 + y^2 = 100$

$m = \frac{7-0}{1-0} = 7$

$y = mx + b$

$7 = 7(1) + b$

$0 = b$

$y = 7x$

Therefore, $7x - y = 0$ or $y = 7x$ is the required equation.

c. The line has a slope of 7; chord MN has a slope of:

$m = \frac{8-6}{-6-8} = \frac{2}{-14} = -\frac{1}{7}$

$7 \times -\frac{1}{7} = -1$

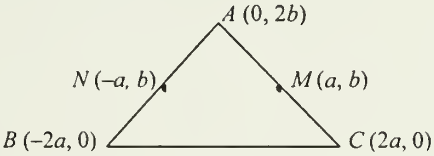
Therefore, the line and chord MN are perpendicular.

Question:

3. Show that the two medians from the base vertices of an isosceles triangle are equal in length.

Solution:

3.



$$M = \left[\frac{0+2a}{2}, \frac{2b+0}{2}\right] = (a, b)$$

$$N = \left[\frac{0+(-2a)}{2}, \frac{2b+0}{2}\right] = (-a, b)$$

$BM^2 = [(a - (-2a))^2 + (b - 0)^2] = 9a^2 + b^2$

$CN^2 = [(2a - (-a))^2 + (b - 0)^2] = 9a^2 + b^2$

As $BM^2 = CN^2$,

$BM = CN$, and so the medians are equal in length.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Specific Outcome

5.5 Verify and prove assertions in plane geometry, using coordinate geometry and trigonometric ratios as necessary. [C, R, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

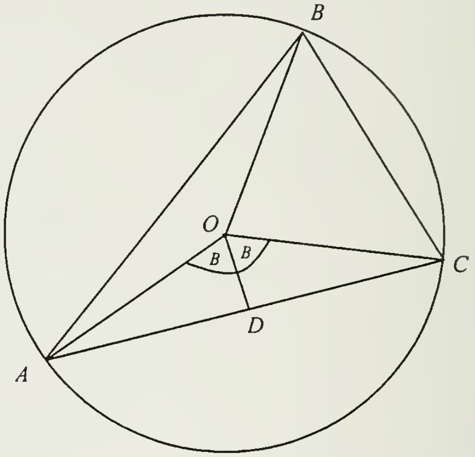
(continued)

Question:

4. Prove that for any triangle inscribed in a circle, the diameter d is equal to either $\frac{a}{\sin A}$, $\frac{b}{\sin B}$ or $\frac{c}{\sin C}$.

Solution:

4. Proof:



Triangle OAC is isosceles, with the vertical angle equal to $2B$ (central angle AOC is double the circumference angle ABC).

Triangles AOD and COD are both right-angled triangles.

So for the triangle AOD, we have $\frac{b}{2R}$ $\sin B$, *or* $d = \frac{b}{\sin B}$.

In the same way, using triangle AOB, we have $d = \frac{c}{\sin C}$.

In the same way, using triangle COB, we have $d = \frac{a}{\sin A}$.

Although the sine law was derived in terms of triangle areas, this example shows that it is also related to the diameter of the circumscribed circle.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• provide a complete solution to question 1 • complete the proof in question 3, given the diagram and the coordinates of A, B and C • provide a paraphrase for a given proof in question 4.	• provide a complete solution to question 2 • construct a proof in question 3, supplying his/her own diagram • construct a proof in question 4, with the diagram given.

STANDARDS IN FINANCE

GENERAL OUTCOME

- Solve consumer problems, using arithmetic operations.

SPECIFIC OUTCOMES

6.1 Solve consumer problems, that may include such examples as:
  - wages earned in various situations
  - property taxation
  - exchange rates.

[CN, E, PS, R, T]

6.2 Reconcile financial statements that may include such examples as:
  - cheque books with bank statements
  - credit card or loan statements.

[CN, PS, T]

6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

6.4 Plot and describe financial data of exponential form. [C, T, V]

6.5 Solve investment and credit problems involving simple and compound interest. [CN, PS, T]

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.1 Solve consumer problems, that may include such examples as:
- wages earned in various situations
- property taxation
- exchange rates.

[CN, E, PS, R, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 4.1.
- This specific outcome lends itself to a project approach; applied mathematics resources have many examples of projects that reflect this specific outcome.
- Specific outcomes in Career and Life Management 20 have some commonality with this specific outcome.
- In McGraw-Hill Ryerson *Mathpower 11*, parts of this specific outcome are covered in the "Getting Started" and "Investigating Math" sections.
- There are two approaches to determining Canada Pension Plan (CPP) and Employment Insurance (EI) deductions—one can use either tables or percentage rates. Tates and maximum values change each year.
- The solution to Question 3 uses 2000 rates for CPP and EI. Later rates are higher for CPP and lower for EI.
- In Addison-Wesley *Mathematics 11*, material on exchange rates is found in an example on page 52 and in questions on page 56.
- Currency exchange rates are updated daily in the media.
- The vocabulary associated with taxation should be reviewed.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete the calculations and present the information appropriately • communicate the mathematics in appropriate everyday terms; some details may be missing • calculate deductions but may not recognize when CPP and EI are at maximum • calculate income tax when the taxable income is in the first tax bracket.	• complete the calculations and present the information in an exemplary manner • communicate the mathematics in a clear way, using numbers, diagrams and appropriate mathematical terms • calculate income tax for any taxable income • calculate deductions when CPP and EI are at maximum.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.1 Solve consumer problems, that may include such examples as:
- wages earned in various situations
- property taxation
- exchange rates.

[CN, E, PS, R, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Fred worked 52 hours last week. He is paid $6.50 per hour for the first 40 hours and time and a half for overtime.
   a. Calculate Fred's gross earnings for the week.
   b. Calculate his deductions, using the following rates: CPP at 3.2%, EI at 2.7%, income tax at 11% of pay after CPP and EI have been deducted.
   c. Calculate Fred's net pay.

Solution:

1. a. Earnings = 40 h × $6.50/h + 12 h × (1.5 × $6.50/h)
   = $377.00

   b. CPP = $377.00 × 0.032 = $12.06
   EI = $377.00 × 0.027 = $10.18
   Income tax = 0.11 × [$377.00 – ($12.06 + $10.18)] = $39.02

   c. Net pay = $377.00 – ($12.06 + $10.18 + $39.02)
   = $315.74

Question:

2. A tourist travelling to New York bought $500 US two weeks before leaving. At the time, $1 CDN was worth $0.67 US. On the day of the trip, the tourist bought an additional $500 US when $1 CDN was worth $0.69 US.
   a. Determine how much the $1000 US cost in Canadian dollars, disregarding any transaction fees.
   b. If the tourist had known exchange rates in advance, when should the currency have been bought? Explain your answer.

Solution:

2. a. $\frac{\$500}{0.67}$ = $746.27 Canadian $\frac{\$500}{0.69}$ = $724.64 Canadian
   Cost = $746.27 + $724.64 = $1470.91 Canadian

   b. The tourist should have bought the $1000 US at the $0.69 US rate. It would have cost $21.63 Canadian less.

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.1 Solve consumer problems, that may include such examples as:
- wages earned in various situations
- property taxation
- exchange rates.

[CN, E, PS, R, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

3. For salaries between $30 000 and $60 000 the approximate rates of income tax are as follows:
   - 0% on the first $9000
   - 25% on the next $20 000
   - 40% on the rest.

   Canada Pension Plan deductions are 3.5% of the first $36 000, and Employment Insurance deductions are 2.4% of the first $39 000.

   Mary earns $47 000 and pays $4000 into an RRSP. What is her monthly take-home pay?

Solution:

3. CPP deduction = 3.5% of $36 000
= $1260
EI deduction = 2.4% of $39 000
= $936
Taxable income = $47 000 – $4000
= $43 000
Tax deduction = 0% of $900 + 25% of $20 000 + 40% of $14 000
= $10 600
Annual take-home = $47 000 – $1260 – $936 – $10 600 – $4000
= $30 204
Monthly take-home = $2517

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete all parts of questions 1 and 2 • show a workable strategy for question 3.	• complete question 3.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.2 Reconcile financial statements that may include such examples as:
- cheque books with bank statements
- credit card or loan statements.

[CN, PS, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 4.2.
- Specific outcomes in Career and Life Management 20 have some commonality with this specific outcome.
- Both Addison-Wesley *Mathematics 11* and McGraw-Hill Ryerson *Mathpower 11* address this specific outcome. *Mathematics 11* addresses it under "Mathematics File," and *Mathpower 11* addresses it under "Investigating Math."
- Credit card statements are more challenging examples than loan statements, as the monthly repayments are not fixed.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• perform the necessary calculations to reconcile a given financial statement, and explain the processes involved • explain the entries on a financial statement • demonstrate an awareness of bank charges; e.g., overdraft protection fee, interest, service charges, automated teller machine (ATM) charges • calculate credit charges and balances due on credit card statements.	• determine which bank plan is the most economical for a given situation.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.2 Reconcile financial statements that may include such examples as:
- cheque books with bank statements
- credit card or loan statements.

[CN, PS, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. a. Reconcile the following cheque book record with the accompanying bank statement.

Cheque book record:

Cheque number	Date	Description of Transaction	Withdrawal	Deposit	Balance
					$380.57
457	Sept 1	Car Payment	$187.50		
458	Sept 10	Car insurance—to Dad	$50.00		
	Sept 12	Gas deposit—from Joe		$12.00	
	Sept 11	Cash withdrawal	$40.00		
	Sept 15	Paycheque		$144.88	
459	Sept 18	Gift for Maxine	$18.77		
460	Sept 20	Registration fee—school	$50.00		
461	Sept 20	Phone bill	$18.45		
	Sept 22	Cash for weekend	$50.00		
	Sept 30	Paycheque		$144.88	
462	Sept 30	New tire for truck	$63.25		

Bank statement:

Date	Transaction Type	Debit	Credit	Balance
	Balance forward			$380.57
Sept 7	Cheque 457	$187.50		$193.07
Sept 11	Cash withdrawal	$40.00		$153.07
Sept 12	Deposit		$12.00	$165.07
Sept 15	Deposit		$144.88	$309.95
Sept 22	Cash withdrawal	$50.00		$259.95
Sept 25	Cheque 461	$18.45		$241.50
Sept 26	Cheque 459	$18.77		$222.73
Sept 30	Deposit		$144.88	$367.61
Sept 30	Service charge	$2.75		$364.86
Sept 30	Maintenance Charge	$1.00		$363.86

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.2 Reconcile financial statements that may include such examples as:
- cheque books with bank statements
- credit card or loan statements.

[CN, PS, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

1. b. The bank provides the following service charge alternatives to the person with the account above:
Plan A: $8.50 per month flat fee
Plan B: $1.00 per month maintenance fee
$0.65 per cheque
$0.40 per ATM transaction.
Which plan would be the most economical for this situation?

c. When would this not be the best alternative?

Solution:

1. a. Cheque book record:

Cheque number	Date	Description of Transaction	Withdrawal	Deposit	Balance
					$380.57
457	Sept 1	Car Payment	187.50		193.07
458	Sept 10	Car insurance—to Dad	50.00		143.07
	Sept 12	Gas deposit—from Joe		12.00	155.07
	Sept 11	Cash withdrawal	40.00		115.07
	Sept 15	Paycheque		144.88	259.95
459	Sept 18	Gift for Maxine	18.77		241.18
460	Sept 20	Registration fee—school	50.00		191.18
461	Sept 20	Phone bill	18.45		172.73
	Sept 22	Cash for weekend	50.00		122.73
	Sept 30	Paycheque		144.88	267.61
462	Sept 30	New tire for truck	63.25		204.36

Reconciliation: Take the statement balance and subtract any outstanding debits and add any outstanding credits to get an adjusted statement balance. Take your cheque register balance and subtract the service charges. This balance should be the same as the adjusted statement balance.

For this example:
Statement: $363.86 – $50.00 – $50.00 – $63.25 = $200.61
Cheque Register: $204.36 – $2.75 – $1.00 = $200.61

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.2 Reconcile financial statements that may include such examples as:
- cheque books with bank statements
- credit card or loan statements.

[CN, PS, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution:

1. b.

Plan A:	$8.50	
Plan B:	6 cheques @ $0.65	$3.90
	2 ATM withdrawals @ $0.40	0.80
	3 ATM deposits @ $0.40	1.20
	Maintenance fee	1.00
		$6.90

This assumes that there is a charge for depositing money through an ATM. Most banks do not charge for this, and an answer of $5.70 is acceptable. In any case, Plan B would be the most economical.

c. Plan A would become more economical if you regularly wrote more than 11 cheques, made more than 18 ATM transactions, or did a combination of these things that would exceed $7.50.

Question:

2. Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

Month	Previous Balance –	Payment Made +	Purchases Charged ⇒	Balance Due +	Credit Charges ⇒	New Balance
February	$314.65	$100.00	$193.75		$5.72	$414.12
March		$150.00	$ 59.60			
April		$140.00	$421.83			$618.62
May	$618.62	$200.00	$ 39.65			
June		$250.00	$ 58.11			
July		$150.00	$ 77.21			

Solution:

2.

Month	Previous Balance –	Payment Made +	Purchases Charged ⇒	Balance Due +	Credit Charges ⇒	New Balance
February	$314.65	$100.00	$193.75	$408.40	$5.72	$414.12
March	$414.12	$150.00	$ 59.60	$323.72	$4.53	$328.25
April	$328.25	$140.00	$421.83	$610.08	$8.54	$618.62
May	$618.62	$200.00	$ 39.65	$458.27	$6.41	$464.68
June	$464.68	$250.00	$ 58.11	$272.79	$3.81	$276.60
July	$276.60	$150.00	$ 77.21	$203.81	$2.84	$206.65

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1, parts a and b • complete question 2.	• justify when it would be more economical to go to Plan A in question 1, part c.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 4.3.
- Consider budget situations that involve only a few variables. This specific outcome could easily expand into a course itself.
- Specific outcomes in Career and Life Management 20 have some commonality with this specific outcome.
- Computer budget programs would be excellent tools for developing budgets.
- This specific outcome lends itself to the use of spreadsheets but can easily be accomplished using the table function of the graphing calculator or using paper and pencil.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• construct a budget • draw and interpret graphs for the purpose of displaying budget data—limited to circle graphs, bar graphs, pictographs • make predictions based on given data • adapt a budget to reflect a single change in situation.	• adapt a budget to multiple changes in situation.

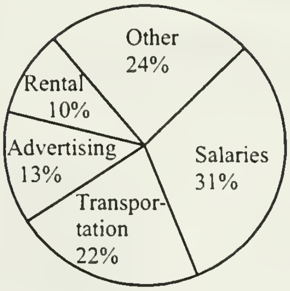
General and Specific Outcomes
General Outcome Solve consumer problems, using arithmetic operations. **Specific Outcome** 6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. Monthly Budget for a Hockey Club



From the circle graph, create a table showing the cost of each expense if $3 000 000 is available in total.

Solution:

1. Salaries $= 0.31 \times 3\ 000\ 000$
Transportation $= 0.22 \times 3\ 000\ 000$
Advertising $= 0.13 \times 3\ 000\ 000$
Rental $= 0.10 \times 3\ 000\ 000$
Other $= 0.24 \times 3\ 000\ 000$

Expense	Cost
Salaries	$930 000
Transportation	$660 000
Advertising	$390 000
Rental	$300 000
Other	$720 000

Question:

2. Yasha completed a class project on the expenses associated with moving away from home and into her own apartment. The following is a summary of her findings:

Item	Total Cost
Rent	$600.00
Utilities	$48.00
Food	$175.00/person
Cable TV	$25.00
Renter's insurance	$35.00
Telephone	$22.50
Transportation	$110.00/person
Clothing/personal	$120.00/person

a. Assuming that she will have one room-mate to share expenses, draw a circle graph to depict her costs in each category.

b. If Yasha had another room-mate move in, what expenses would change for her? Adjust the figures in your chart to reflect these changes, and redraw your circle graph to depict the new costs.

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

- ☐ Conceptual
- ☑ Procedural
- ☑ Problem-solving

(continued)

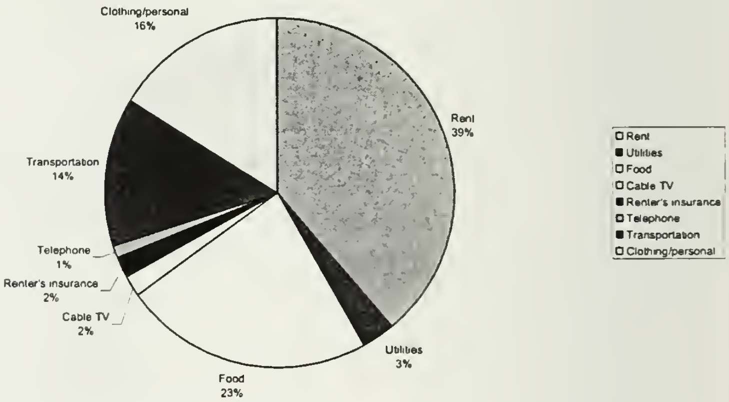
Solution:

2. a.

Item	Total Cost	Yasha's Cost	Per Cent	Degrees
Rent	$600.00	$300.00	39	140
Utilities	$48.00	$24.00	3	11
Food	$350.00	$175.00	23	83
Cable TV	$25.00	$12.50	2	7
Renter's insurance	$35.00	$17.50	2	7
Telephone	$22.50	$11.25	1	4
Transportation	$220.00	$110.00	14	50
Clothing/personal	$240.00	$120.00	16	58
		$770.25	100%	360

Either percentage or degrees can be used to define the sectors of the circle graph.

Yasha's Costs



b. The new table is as follows:

Item	Total Cost	Yasha's Cost	Per Cent	Degrees
Rent	$600.00	$200.00	31	112
Utilities	$48.00	$16.00	2	7
Food	$525.00	$175.00	27	97
Cable TV	$25.00	$8.33	1	4
Renter's insurance	$35.00	$11.67	2	7
Telephone	$22.50	$7.50	1	4
Transportation	$330.00	$110.00	17	61
Clothing/personal	$360.00	$120.00	19	68
		$648.50	100%	360

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

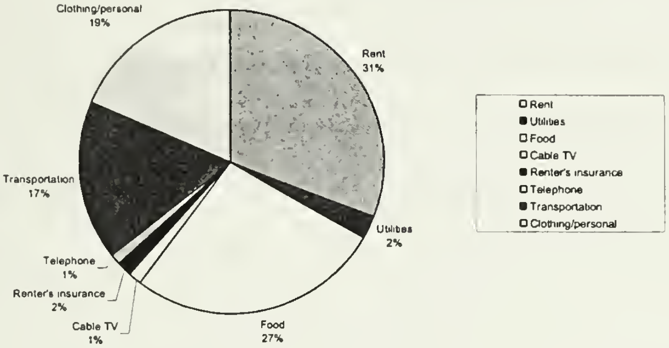
[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

2. b.

Expenses with Two Room-mates



Question:

3. Imagine yourself as a student who will graduate from a post-secondary program next year. You expect to be offered a permanent job and have decided you will buy a car and live independently in an apartment. As a student, you have been sharing an apartment with two room-mates and have been using public transportation. You will also have a student loan to pay back when you graduate. Develop two budgets: one to reflect your expenses as a student, another to reflect your expenses as a worker with a full-time job. Explain the choices you make in each budget.

Solution:

3. Answers will vary, but should reflect current costs and salaries and may or may not include deductions from salaries. A sample answer is:

Expense Category	Expenses per month
Rent (one-third share)	$175
Food (at home and out)	$180
Clothing/personal	$110
Transportation (bus)	$90
Renter insurance	$25
Student loan expenses	$5
Tuition and books	$350
Deductions from part-time job	$60
Total	$995

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

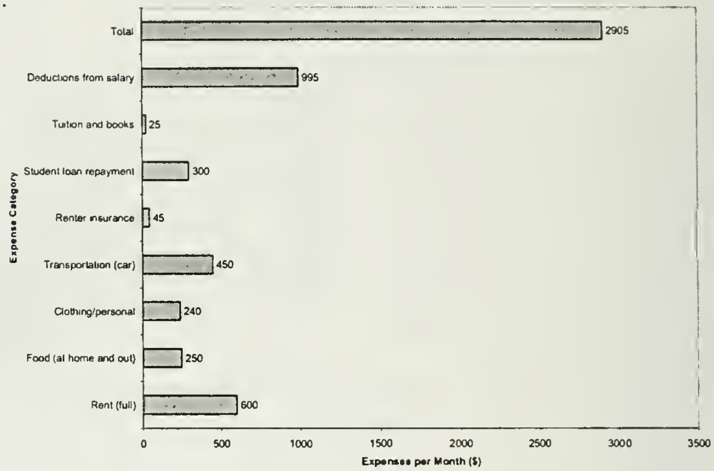
[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

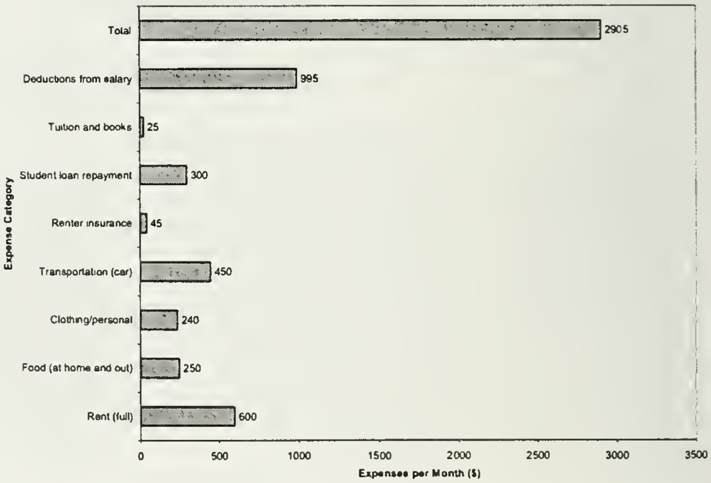
3.

Student Budget



Expense Category	Expenses per month
Rent (full)	$600
Food (at home and out)	$250
Clothing/personal	$240
Transportation (car)	$450
Renter insurance	$45
Student loan repayment	$300
Tuition and books	$25
Deductions from salary	$995
Total	$2905

Working Budget



(continued)

General and Specific Outcomes
General Outcome Solve consumer problems, using arithmetic operations. **Specific Outcome** 6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

3.

Explanations:

- car expenses are based on a used car costing under $10 000, as the average starting salary will not be enough to afford a new car
- student loan repayment is based on repaying $20 000 over 7 years
- clothing allowance is higher for a working person compared to a student
- food allowance is higher, as there may be a need to eat out more often as a working person
- no money is set aside for savings or investments

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1 • translate the numbers in the table in question 2 into percentages or degrees, and show them graphically in question 2, part a • identify what items would change in question 2, part b; make the necessary changes in the data; and complete the corresponding calculations and graph in question 2, part b • produce a student budget in question 3.	• produce a working budget in question 3, and explain the choices made.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.4 Plot and describe financial data of exponential form. [C, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome has some commonality with Applied Mathematics 20 specific outcome 2.3.
- This specific outcome could be incorporated with specific outcome 6.5.
- Data should be restricted to financial data, such as car depreciation and RRSP growth.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate, plot and describe the characteristics of the data from a given scenario • analyze and interpret trends in data presented in graphical form • extrapolate or interpolate, given a set of data that models exponential growth.	• make predictions based on changes to the parameters within the scenario.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.4 Plot and describe financial data of exponential form. [C, T, V]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

Question:

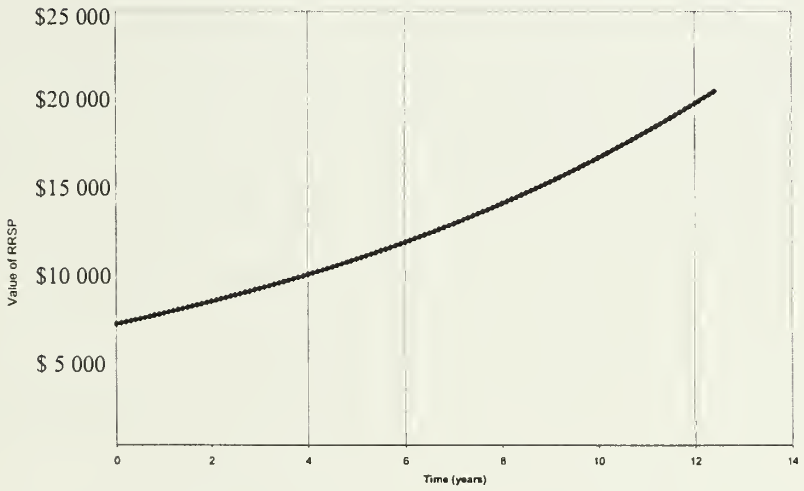
1. An initial amount of $7000 is invested in an RRSP. The growth of the value is as follows.

Time (years)	Value
0	$7 000
1	$7 630
2	$8 316
3	$9 065
4	$9 881
5	$10 770

a. Plot this data on a graph.
b. Estimate the time needed for the RRSP to reach $14 000.
c. Determine the approximate value of the RRSP after 12 years.

Solution:

1. a.



Growth of a $7000 RRSP

b. Based on extrapolation, it will take approximately 8 years for the value to reach $14 000.
c. After 12 years, the RRSP will be worth approximately $19 700.

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.4 Plot and describe financial data of exponential form. [C, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

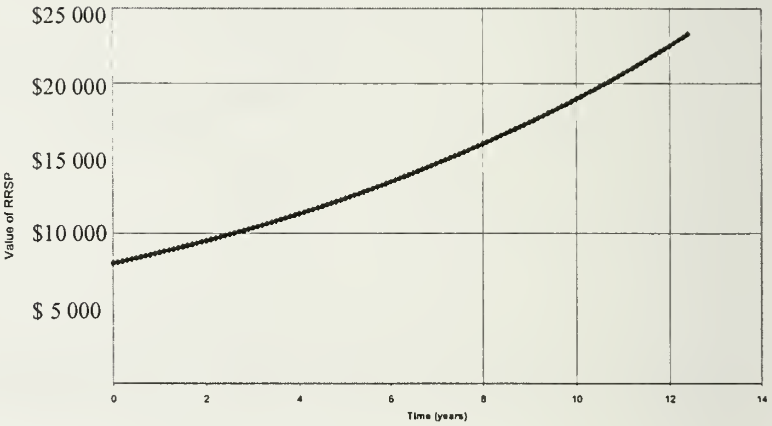
2. Suppose an initial RRSP investment of $8000 is earning interest at the same rate as in question 1.
   a. Generate a table and graph the data.
   b. Compare this graph to the graph in question 1.
   c. Write a general statement of what happens when you change the initial contribution.
   d. Predict what would happen to the graph if you changed the rate of interest but kept the initial contribution the same, and relate these changes to the difference in the amount of interest earned each year.

Solution:

2. a.

Time (years)	Value
0	$8 000.00
1	$8 720.00
2	$9 504.80
3	$10 360.23
4	$11 292.65
5	$12 308.99

Growth of a $8000 RRSP



(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.4 Plot and describe financial data of exponential form. [C, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution:

2. b. Compared to the graph in question 1, this graph has a steeper slope.
   c. When the initial contribution is increased, the interest accumulates more quickly and the value increases more quickly.
   d. If the rate of interest was less, the graph would be less steep. The difference in the amount of interest earned each year increases more slowly. If the rate of interest was greater, the graph would be steeper. The difference in the amount of interest earned each year increases more rapidly.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide a complete solution to question 1 and question 2, parts a, b and c • develop a strategy for completing question 2, part d, but may have difficulty providing the full explanation.	• provide a complete solution to question 2, part d.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

6.5 Solve investment and credit problems involving simple and compound interest. [CN, PS, T]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 20 specific outcome 4.4.
- This specific outcome lends itself to using the graphing calculator, in particular the built-in finance functions.
- Students should be able to use $A = P(1+i)^n$ to solve for A, P, i and n.
- A variety of methods can be used to evaluate i and n, including guess and check, estimating from a graph, spreadsheet analysis or financial functions on a calculator. The use of logarithms is a Pure Mathematics 30 specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve for A, P, i and n, using financial functions on a calculator • change units for i and n as required.	• solve for A and P algebraically • use a logical approach to produce reasonable values for i and n.

General and Specific Outcomes
General Outcome Solve consumer problems, using arithmetic operations. **Specific Outcome** 6.5 Solve investment and credit problems involving simple and compound interest. [CN, PS, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. How much money must be invested today at 6.5%/a interest, compounded semiannually, in order to have $1000 five years from now?

Solution:

1. Algebraic solution:

$$A = P(1+i)^n \qquad i = \frac{0.065}{2}$$

$$1000 = P(1.0325)^{5\times 2} \qquad = 0.0325$$

$$\frac{1000}{(1.0325)^{10}} = P$$

$$\$726.27 = P$$

Financial function solution:
The amount is $726.27.
[N = 10, PMT = 0, FV = 1000, P/Y = 2, C/Y = 2, BEGIN]

Question:

2. A bank offers a $100 000 mortgage amortized over 20 years at 6.75%/a, compounded semiannually.
   a. Determine the monthly payment.
   b. Determine the total amount to be paid, including the interest.
   c. If another mortgage is offered at 6.45% instead of 6.75%, determine the monthly payment and total to be paid.
   d. What would change, if the mortgages were amortized over 15 years?

Solution:

2. Using the financial program on a graphing calculator:
   a. The monthly payment is $754.84. [PV = –100 000, N = 240, FV = 0, P/Y = 12, C/Y = 2]
   b. The total paid will be $181 162.56.
   c. Change the interest to 6.45% and then at 6.45% the monthly payment would be $737.65 and the total paid would be $177 036.00.
   d. Change N to 180 and the monthly payments would be: $879.76 at 6.75%, $863.70 at 6.45%
      Total amount paid would be: $155 466.00 at 6.45%,
      $158 356.80 at 6.75%.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete all parts of questions 1 and 2, using financial applications software.	• complete question 1 algebraically.